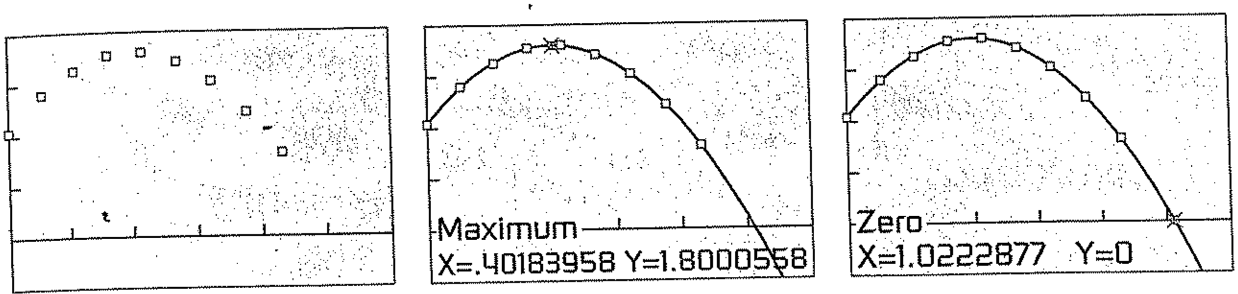


[0, 1.2] by [−0.5, 2.0]
(a)

[0, 1.2] by [−0.5, 2.0]
(b)

[0, 1.2] by [−0.5, 2.0]
(c)

Figure 2.12 Scatter plot and graph of height versus time for Example 8.

The curve in Figure 2.12b appears to fit the data extremely well, and $R^2 \approx 0.999$. You may have noticed, however, that Table 2.3 contains the ordered pair (0.4300, 1.80392) and that $1.80392 > 1.800$, which is the maximum shown in Figure 2.12b. So, even though our model is theoretically based and an excellent fit to the data, it is not a perfect model. Despite its imperfections, the model provides accurate and reliable predictions about the CBR experiment.

Quick Review 2.1

In Exercises 1–2, write an equation in slope-intercept form for a line with the given slope m and y-intercept b.

1. $m = 8, \quad b = 3.6$ **2.** $m = -1.8, \quad b = -2$

In Exercises 3–4, write an equation for the line containing the given points. Graph the line and points.

3. $(-2, 4)$ and $(3, 1)$ **4.** $(1, 5)$ and $(-2, -3)$

In Exercises 5–8, expand the expression.

5. $(x + 3)^2$ **6.** $(x - 4)^2$

7. $3(x - 6)^2$ **8.** $-3(x + 7)^2$

In Exercises 9–10, factor the trinomial.

9. $2x^2 - 4x + 2$ **10.** $3x^2 + 12x + 12$

Section 2.1 Exercises

In Exercises 1–6, determine which are polynomial functions. For those that are, state the degree and leading coefficient. For those that are not, explain why not.

1. $f(x) = 3x^{-5} + 17$ **2.** $f(x) = -9 + 2x$

3. $f(x) = 2x^5 - \frac{1}{2}x + 9$ **4.** $f(x) = 13$

5. $h(x) = \sqrt[3]{27x^3 + 8x^6}$ **6.** $k(x) = 4x - 5x^2$

In Exercises 7–12, write an equation for the linear function f satisfying the given conditions. Graph $y = f(x)$.

7. $f(-5) = -1$ and $f(2) = 4$

8. $f(-3) = 5$ and $f(6) = -2$

9. $f(-4) = 6$ and $f(-1) = 2$

10. $f(1) = 2$ and $f(5) = 7$

11. $f(0) = 3$ and $f(3) = 0$

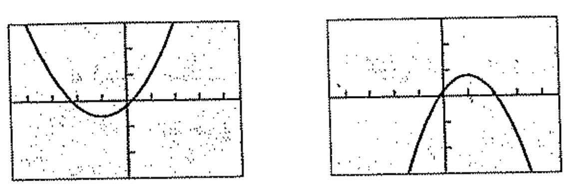
12. $f(-4) = 0$ and $f(0) = 2$

In Exercises 13–18, match a graph to the function. Explain your choice. (All graphs are shown in the same viewing window.)

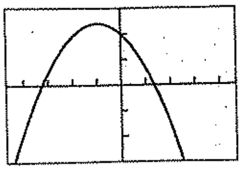
13. $f(x) = 2(x + 1)^2 - 3$ **14.** $f(x) = 3(x + 2)^2 - 7$

15. $f(x) = 4 - 3(x - 1)^2$ **16.** $f(x) = 12 - 2(x - 1)^2$

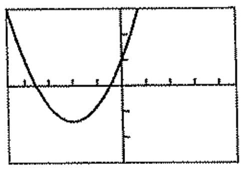
17. $f(x) = 2(x - 1)^2 - 3$ **18.** $f(x) = 12 - 2(x + 1)^2$



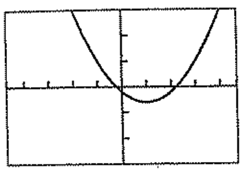
(a) (b)



(c)



(d)



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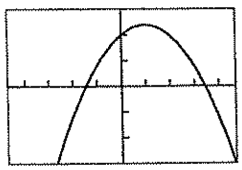


(f)

In Exercises 19–28, find the vertex and axis of the graph of the function. Support your answer graphically.

19. $f(x) = 3(x - 1)^2 + 5$ **20.** $g(x) = -3(x + 2)^2 - 1$

21. $f(x) = 5(x - 1)^2 - 7$ **22.** $g(x) = 2(x - \sqrt{3})^2 + 4$

23. $f(x) = 3x^2 + 5x - 4$ **24.** $f(x) = -2x^2 + 7x - 3$

25. $f(x) = 8x - x^2 + 3$ **26.** $f(x) = 6 - 2x + 4x^2$

27. $g(x) = 5x^2 + 4 - 6x$ **28.** $h(x) = -2x^2 - 7x - 4$

In Exercises 29–34, use algebraic methods to describe the graph of the function. Sketch the graph by hand. Support your answers graphically.

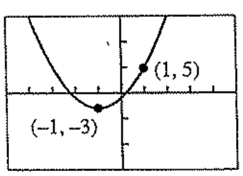
29. $f(x) = x^2 - 4x + 6$ **30.** $g(x) = x^2 - 6x + 12$

31. $f(x) = 10 - 16x - x^2$ **32.** $h(x) = 8 + 2x - x^2$

33. $f(x) = 2x^2 + 6x + 7$ **34.** $g(x) = 5x^2 - 25x + 12$

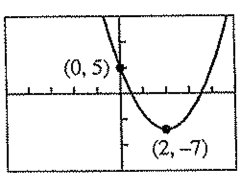
In Exercises 35–38, write an equation for the parabola shown, using the fact that one of the given points is the vertex.

35.



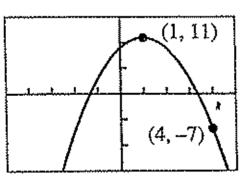
[−5, 5] by [−15, 15]

36.



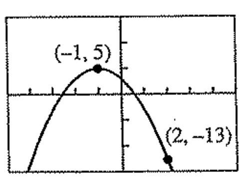
[−5, 5] by [−15, 15]

37.



[−5, 5] by [−15, 15]

38.



[−5, 5] by [−15, 15]

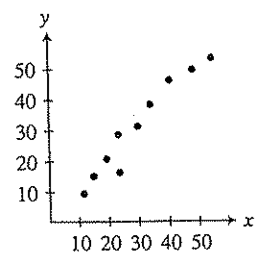
In Exercises 39 and 40, write an equation for the quadratic function whose graph contains the given vertex and point.

39. Vertex (1, 3), point (0, 5)

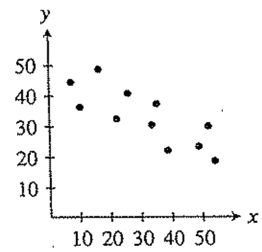
40. Vertex (−2, −5), point (−4, −27)

In Exercises 41–44, describe the strength and direction of the linear correlation.

41.



42.



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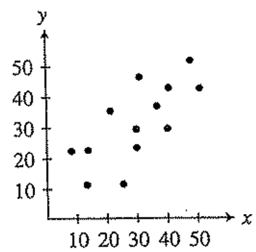

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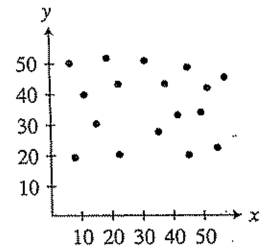


45. Comparing Age and Weight A group of male children were weighed. Their ages and weights are recorded in Table 2.4.

(a) Draw a scatter plot of these data.

(b) **Writing to Learn** Describe the strength and direction of the correlation between age and weight.

Table 2.4 Children's Age and Weight

Age (months)	Weight (pounds)
18	23
20	25
24	24
26	32
27	33
29	29
34	35
39	39
42	44

46. Life Expectancy Table 2.5 shows the average number of additional years a U.S. citizen is expected to live for various ages.

(a) Draw a scatter plot of these data.

(b) **Writing to Learn** Describe the strength and direction of the correlation between age and life expectancy.

Table 2.5 U.S. Life Expectancy

Age (years)	Life Expectancy (years)
10	66
20	56
30	47
40	37
50	29
60	20
70	14
80	8

Source: U.S. National Center for Health Statistics, Vital Statistics of the United States.

47. **Straight-Line Depreciation** MaiLee bought a computer for her home office and depreciated it over 5 years using the straight-line method. If its initial value was \$2350, what is its value 3 years later?

48. **Costly Doll Making** Patrick's doll-making business has weekly fixed costs of \$350. If the cost for materials is \$4.70 per doll and his total weekly costs average \$500, about how many dolls does Patrick make each week?

49. Table 2.6 shows the mean annual compensation of construction workers. Let $x = 0$ stand for 1990, $x = 1$ for 1991, and so forth.

Table 2.6 Construction Worker Average Annual Compensation

Year	Annual Compensation (dollars)
1990	33,701
1994	37,275
1995	37,417
1996	38,456

Source: U.S. Bureau of Economic Analysis, Statistical Abstract of the United States, 1998.

(a) Find the linear regression model for the data. What does the slope in the regression model represent?

(b) Use the linear regression model to predict the construction worker average annual compensation in the year 2002.

50. **Total Revenue** The per unit price p (in dollars) of a popular toy when x units (in thousands) are produced is modeled by the function

$$\text{price} = p = 12 - 0.025x.$$

The revenue (in thousands of dollars) is the product of the price per unit and the number of units (in thousands) produced. That is,

$$\text{revenue} = xp = x(12 - 0.025x).$$

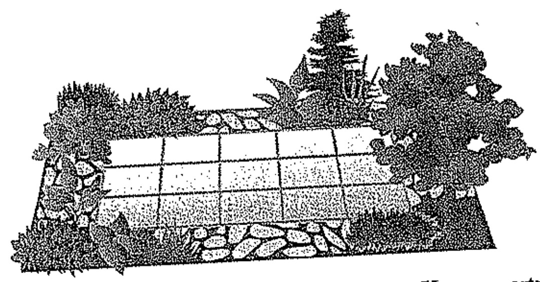
(a) State the dimensions of a viewing window that shows a graph of the revenue model for producing 0 to 100,000 units.

(b) How many units should be produced if the total revenue is to be \$1,000,000.

51. **Finding Maximum Area** Among all the rectangles whose perimeters are 100 ft, find the dimensions of the one with maximum area.

52. **Finding the Dimensions of a Painting** A large painting in the style of Rubens is 3 ft longer than it is wide. If the wooden frame is 12 in. wide, the area of the picture and frame is 208 ft^2, find the dimensions of the painting.

53. **Using Algebra in Landscape Design** Julie Stone designed a rectangular patio that is 25 ft by 40 ft. This patio is surrounded by a terraced strip of uniform width planted with small trees and shrubs. If the area A of this terraced strip is 504 ft^2, find the width x of the strip.



54. **Management Planning** The Welcome Home apartment rental company has 1600 units available, of which 800 are currently rented at \$300 per month. A market survey indicates that each \$5 decrease in monthly rent will result in 20 new leases.

(a) Determine a function $R(x)$ that models the total rental income realized by Welcome Home, where x is the number of \$5 decreases in monthly rent.

(b) Find a graph of $R(x)$ for rent levels between \$175 and \$300 (that is, $0 \le x \le 25$) that clearly shows a maximum for $R(x)$.

(c) What rent will yield Welcome Home the maximum monthly income?

55. **Group Activity Beverage Business** The Sweet Drip Beverage Co. sells cans of soda pop in machines. It finds that sales average 26,000 cans per month when the cans sell for 50¢ each. For each nickel increase in the price, the sales per month drop by 1000 cans.

(a) Determine a function $R(x)$ that models the total revenue realized by Sweet Drip, where x is the number of \$0.05 increases in the price of a can.

(b) Find a graph of $R(x)$ that clearly shows a maximum for $R(x)$.

(c) How much should Sweet Drip charge per can to realize the maximum revenue? What is the maximum revenue?

56. **Group Activity Sales Manager Planning** Jack was named District Manager of the Month at the Sylvania Wire Co. due to his hiring study. It shows that each of the 30 salespersons he supervises average \$50,000 in sales eac month, and that for each additional salesperson he would

hire, the average sales would decrease $1000 per month. Jack concluded his study by suggesting a number of salespersons that he should hire to maximize sales. What was that number?

57. **Baseball Throwing Machine** The Sandusky Little League uses a baseball throwing machine to help train 10-year-old players to catch high pop-ups. It throws the baseball straight up with an initial velocity of 48 ft/sec from a height of 3.5 ft.

(a) Find an equation that models the height of the ball t seconds after it is thrown.

(b) What is the maximum height the baseball will reach? How many seconds will it take to reach that height?

58. **Fireworks Planning** At the Bakersville Fourth of July celebration, fireworks are shot by remote control into the air from a pit that is 10 ft below the earth's surface.

(a) Find an equation that models the height of an aerial bomb t seconds after it is shot upwards with an initial velocity of 80 ft/sec. Graph the equation in both function mode and parametric mode.

(b) What is the maximum height above ground level that the aerial bomb will reach? How many seconds will it take to reach that height?

59. **Landscape Engineering** In her first project after being employed by Land Scapes International, Becky designs a decorative water fountain that will shoot water to a maximum height of 48 ft. What should be the initial velocity of each drop of water to achieve this maximum height? (*Hint*: Use a grapher and a guess-and-check strategy.)

60. **Patent Applications** Using quadratic regression on the data in Table 2.7, predict the year when the number of patent application will reach 300,000. Let $x = 0$ stand for 1980, $x = 1$ for 1981 and so forth.

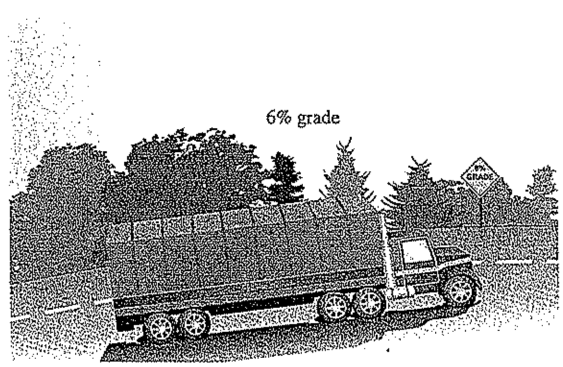
Table 2.7 U.S. Patent Applications

Year	Applications (thousands)
1980	113.0
1985	127.1
1990	176.7
1991	178.4
1992	187.2
1993	189.4
1994	206.9
1995	226.9
1996	211.6

Source: U.S. Bureau of the Census, Statistical Abstract of the United States, 1998 (Washington, D.C., 1998).

Explorations

61. **Highway Engineering** Interstate 70 west of Denver, Colorado, has a section posted as a 6% grade. This means that for a horizontal change of 100 ft there is a 6-ft vertical change.



(a) Find the slope of this section of the highway.

(b) On a highway with a 6% grade what is the horizontal distance required to climb 250 ft?

(c) A sign along the highway says 6% grade for the next 7 mi. Estimate how many feet of vertical change there are along those next 7 mi. (There are 5280 ft in 1 mile.)

62. A group of female children were weighed. Their ages and weights are recorded in Table 2.8.

(a) Draw a scatter plot of the data.

(b) Find the linear regression model.

(c) Interpret the slope of the linear regression equation.

(d) Superimpose the regression line on the scatter plot.

(e) Use the regression model to predict the weight of a 30-month-old girl.

Table 2.8 Children's Ages and Weights

Age (months)	Weight (pounds)
19	22
21	23
24	25
27	28
29	31
31	28
34	32
38	34
43	39

63. Table 2.9 gives the average salaries for NBA players over several seasons.

(a) Draw a scatter plot of the data.

(b) Find the linear regression model.

(c) Interpret the slope of the linear regression equation.

(d) Superimpose the regression line on the scatter plot.

(e) Use the regression model to predict the average salary for NBA players during the 2001–2002 season.

Table 2.9 National Basketball Association Average Player Salaries

Season	Salary
1990–1991	\$1,034,000
1991–1992	\$1,202,000
1992–1993	\$1,348,000
1993–1994	\$1,558,000
1994–1995	\$1,800,000
1995–1996	\$2,027,261
1996–1997	\$2,189,442

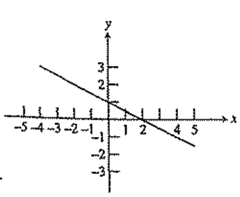
Source: *Paul D. Staudohar, Salary Caps in Team Sports, Compensation and Working Conditions, Spring 1998.*

64. **Writing to Learn Identifying Graphs of Linear Functions**

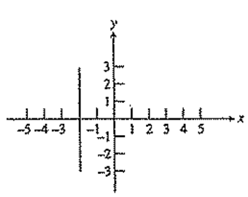
(a) Which of the lines graphed below are graphs of linear functions? Explain.

(b) Which of the lines graphed below are graphs of functions? Explain.

(c) Which of the lines graphed below are not graphs of functions? Explain.



(i)



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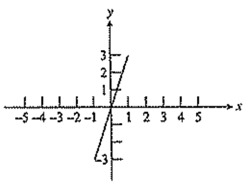

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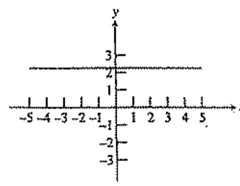

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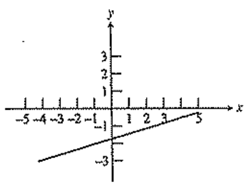

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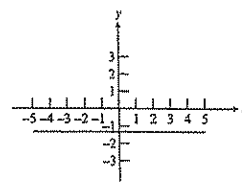


(vi)

65. **Average Rate of Change** Let $f(x) = x^2$, $g(x) = 3x + 2$, $h(x) = 7x - 3$, $k(x) = mx + b$, and $l(x) = x^3$.

(a) Compute the average rate of change of f from $x = 1$ to $x = 3$.

(b) Compute the average rate of change of f from $x = 2$ to $x = 5$.

(c) Compute the average rate of change of f from $x = a$ to $x = c$.

(d) Compute the average rate of change of g from $x = 1$ to $x = 3$.

(e) Compute the average rate of change of g from $x = 1$ to $x = 4$.

(f) Compute the average rate of change of g from $x = a$ to $x = c$.

(g) Compute the average rate of change of h from $x = a$ to $x = c$.

(h) Compute the average rate of change of k from $x = a$ to $x = c$.

(i) Compute the average rate of change of l from $x = a$ to $x = c$.

Extending the Ideas

66. **Minimizing Sums of Squares** The linear regression line is often called the **least squares line** because it minimizes the sum of the squares of the **residuals**, the differences between actual y values and predicted y values:

$$\textit{residual} = y_i - (ax_i + b),$$

where (x_i, y_i) are the given data pairs and $y = ax + b$ is the regression equation, as shown in the figure on the next page.

Quick Review 2.2

In Exercises 1–6, write the following expressions using only positive integer powers.

1. $x^{2/3}$
2. $p^{5/2}$
3. d^{-2}
4. x^{-7}
5. $q^{-4/5}$
6. $m^{-1.5}$

In Exercises 7–10, write the following expressions in the form $k \cdot x^a$ using a single rational number for the power a.

7. $\sqrt{9x^3}$
8. $\sqrt[3]{8x^5}$
9. $\sqrt[3]{\dfrac{5}{x^4}}$
10. $\dfrac{4x}{\sqrt{32x^3}}$

Section 2.2 Exercises

In Exercises 1–10, determine if the function is a power function, given that c, g, k, and π represent constants. For those that are, state the power and constant of variation. For those that are not, explain why not. If function notation is not used, name the independent variable.

1. $f(x) = -\dfrac{1}{2}x^5$
2. $f(x) = 9x^{5/3}$
3. $f(x) = 3 \cdot 2^x$
4. $f(x) = 13$
5. $E(m) = mc^2$
6. $KE(v) = \dfrac{1}{2}kv^5$
7. $d = \dfrac{1}{2}gt^2$
8. $V = \dfrac{4}{3}\pi r^3$
9. $I = \dfrac{k}{d^2}$
10. $F(a) = m \cdot a$

In Exercises 11–16, determine if the function is a monomial function, given that l and π represent constants. For those that are, state the degree and leading coefficient. For those that are not, explain why not. If function notation is not used, name the independent variable.

11. $f(x) = -4$
12. $f(x) = 3 \cdot x^{-5}$
13. $y = -6 \cdot x^7$
14. $y = -2 \cdot 5^x$
15. $S = 4\pi r^2$
16. $A = l \cdot w$

In Exercises 17–22, write the statement as a power function equation.

17. The area A of an equilateral triangle varies directly as the square of the length s of its sides.
18. The volume V of a circular cylinder with fixed height is proportional to the square of its radius r.
19. The current I in an electrical circuit is inversely proportional to the resistance R, with constant of variation V.
20. Charles's Law states, the volume V of an enclosed ideal gas at a constant pressure varies directly as the absolute temperature T.
21. The energy E produced in a nuclear reaction is proportional to the mass m, with the constant of variation being c^2, the square of the speed of light.
22. The speed p of a free-falling object that has been dropped from rest varies as the square root of the distance traveled d, with a constant of variation $k = \sqrt{2g}$.

In Exercises 23–26, write a sentence that expresses the relationship in the formula, using the language of variation or proportion.

23. $w = mg$, where w and m are the weight and mass of an object and g is the constant acceleration due to gravity.
24. $C = \pi D$, where C and D are the circumference and diameter of a circle and π is the usual mathematical constant.
25. $n = c/v$, where n is the refractive index of a medium, v, is the velocity of light in the medium, and c is the constant velocity of light in free space.
26. $d = p^2/(2g)$, where d is the distance traveled by a free-falling object dropped from rest, p is the speed of the object, and g is the constant acceleration due to gravity.

In Exercises 27–28, data are given for y as a power function of x. Write an equation for the power function, and state its power and constant of variation.

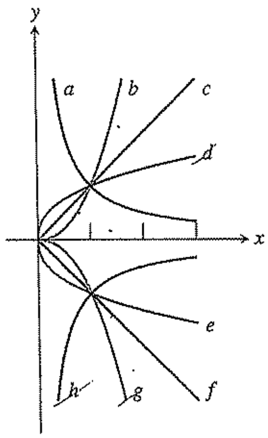
27.

x	2	4	6	8	10
y	2	0.5	0.222...	0.125	0.08

28.

x	1	4	9	16	25
y	−2	−4	−6	−8	−10

In Exercises 29–34, match the equation to one of the curves labeled in the figure.



29. $f(x) = -\frac{2}{3}x^4$
30. $f(x) = \frac{1}{2}x^{-5}$
31. $f(x) = 2x^{1/4}$
32. $f(x) = -x^{5/3}$
33. $f(x) = -2x^{-2}$
34. $f(x) = 1.7x^{2/3}$

In Exercises 35–40, describe how to obtain the graph of the given monomial function from the graph of $g(x) = x^n$ with the same power n. State whether f is even or odd. Sketch the graph by hand and support your answer with a grapher.

35. $f(x) = \frac{2}{3}x^4$
36. $f(x) = 5x^3$
37. $f(x) = -1.5x^5$
38. $f(x) = -2x^6$
39. $f(x) = \frac{1}{4}x^8$
40. $f(x) = \frac{1}{8}x^7$

In Exercises 41–44, state the power and constant of variation for the function, graph it, and analyze it in the manner of Example 2 of this section.

41. $f(x) = 2x^4$
42. $f(x) = -3x^3$
43. $f(x) = \frac{1}{2}\sqrt[4]{x}$
44. $f(x) = -2x^{-3}$

In Exercises 45–50, state the values of the constants k and a for the function $f(x) = k \cdot x^a$. Describe the portion of the curve that lies in Quadrant I or IV. Determine whether f is even, odd, or undefined for $x < 0$. Describe the rest of the curve if any. Graph the function to see whether it matches the description.

45. $f(x) = 3x^{1/4}$
46. $f(x) = -4x^{2/3}$
47. $f(x) = -2x^{4/3}$
48. $f(x) = \frac{2}{5}x^{5/2}$
49. $f(x) = \frac{1}{2}x^{-3}$
50. $f(x) = -x^{-4}$

51. **Boyle's Law** The volume of an enclosed gas (at a constant temperature) varies inversely as the pressure. If the pressure of a 3.46-L sample of neon gas at 302°K is 0.926 atm, what would the volume be at a pressure of 1.452 atm if the temperature does not change?

52. **Charles's Law** The volume of an enclosed gas (at a constant pressure) varies directly as the absolute temperature. If the pressure of a 3.46-L sample of neon gas at 302°K is 0.926 atm, what would the volume be at a temperature of 338°K if the pressure does not change?

53. **Diamond Refraction** Diamonds have the extremely high refraction index of $n = 2.42$ on average over the range of visible light. Use the formula from Exercise 25 and the fact that $c \approx 3.00 \times 10^8$ m/sec to determine the speed of light through a diamond.

54. **Windmill Power** The power P (in watts) produced by a windmill is proportional to the cube of the wind speed v (in mph). If a wind of 10 mph generates 15 watts of power, how much power is generated by winds of 20, 40, and 80 mph? Make a table and explain the pattern.

Explorations

55. **Keeping Warm** For mammals and other warm-blooded animals to stay warm requires quite a bit of energy. Temperature loss is related to surface area, which is related to body weight, and temperature gain is related to circulation, which is related to pulse rate. In the final analysis, scientists have concluded that the pulse rate r of mammals is a power function of their body weight w.

    (a) Draw a scatter plot of the data in Table 2.13.

    (b) Find the power regression model.

    (c) Superimpose the regression curve on the scatter plot.

    (d) Use the regression model to predict the pulse rate for a 450-kg horse. Is the result close to the 38 beats/min reported by A. J. Clark in 1927?

Table 2.13 Weight and Pulse Rate of Selected Mammals

Mammal	Body weight (kg)	Pulse rate (beats/min)
Rat	0.2	420
Guinea pig	0.3	300
Rabbit	2	205
Small dog	5	120
Large dog	30	85
Sheep	50	70
Human	70	72

Source: A. J. Clark, Comparative Physiology of the Heart (New York: Macmillan, 1927).

56. **Light Intensity** Velma and Reggie gathered the data in Table 2.14 using a 100-watt light bulb and a Calculator-Based Laboratory™ (CBL™) with a light-intensity probe.

    (a) Draw a scatter plot of the data in Table 2.14.

    (b) Find the power regression model. Is the power close to the theoretical value of $a = -2$?

    (c) Superimpose the regression curve on the scatter plot.

(d) Use the regression model to predict the light intensity at distances of 1.7 m and 3.4 m.

Table 2.14 Light Intensity Data for a 100-W Light Bulb

Distance (m)	Intensity (W/m^2)
1.0	7.95
1.5	3.53
2.0	2.01
2.5	1.27
3.0	0.90

57. **Group Activity Rational Powers** Working in a group of three students, investigate the behavior of power functions of the form $f(x) = k \cdot x^{m/n}$, where m and n are positive with no factors in common. Have one group member investigate each of the following cases:

- n is even
- n is odd and m is even
- n is odd and m is odd

For each case, decide whether f is even, f is odd, or f is undefined for $x < 0$. Solve graphically and confirm algebraically in a way to convince the rest of your group and your entire class.

Extending the Ideas

58. **Writing to Learn Irrational Powers** A negative number to an irrational power is undefined. Analyze the graphs of $f(x) = x^{\pi}, x^{1/\pi}, x^{-\pi}, -x^{\pi}, -x^{1/\pi}$, and $-x^{-\pi}$. Prepare a sketch of all six graphs on one set of axes, labeling each of the curves. Write an explanation for why each graph is positioned and shaped as it is.

59. **Planetary Motion Revisited** Convert the time and distance units in Table 2.11 to the Earth-based units of years and astronomical units using

$$1 \text{ yr} = 365.2 \text{ days} \quad \text{and} \quad 1 \text{ AU} = 149.6 \text{ Gm}.$$

Use this "re-expressed" data to obtain a power function model. Show algebraically that this model closely approximates Kepler's equation $T^2 = a^3$.

60. **Free Fall Revisited** The **speed** p of an object is the absolute value of its velocity v. The distance traveled d by an object dropped from an initial height s_0 with a current height s is given by

$$d = s_0 - s$$

until it hits the ground. Use this information and the free-fall motion formulas from Section 2.1 to prove that

$$d = \frac{1}{2}gt^2,\ p = gt, \text{ and therefore } p = \sqrt{2gd}.$$

Do the results of Example 6 approximate this last formula?

61. Prove that $g(x) = 1/f(x)$ is even if and only if $f(x)$ is even and that $g(x) = 1/f(x)$ is odd if and only if $f(x)$ is odd.

62. Use the results in Exercise 61 to prove that $g(x) = k \cdot x^{-a}$ is even if and only if $f(x) = k \cdot x^a$ is even and that $g(x) = k \cdot x^{-a}$ is odd if and only if $f(x) = k \cdot x^a$ is odd.

63. **Joint Variation** If a variable z varies as the product of the variables x and y, we say z **varies jointly** as x and y, and we write $z = k \cdot x \cdot y$, where k is the constant of variation. Write a sentence that expresses the relationship in the formula, using the language of joint variation.

(a) $F = m \cdot a$, where F and a are the force and acceleration acting on an object of mass m.

(b) $KE = (1/2)m \cdot v^2$, where KE and v are the kinetic energy and velocity of an object of mass m.

(c) $F = G \cdot m_1 \cdot m_2/r^2$, where F is the force of gravity acting on objects of masses m_1 and m_2 with a distance r between their centers and G is the universal gravitational constant.

2.3 Polynomial Functions of Higher Degree with Modeling

Graphs of Polynomial Functions • End Behavior of Polynomial Functions • Zeros of Polynomial Functions • Intermediate Value Theorem • Modeling

Graphs of Polynomial Functions

As we saw in Section 2.1, a polynomial function of degree 0 is a constant function and graphs as a horizontal line. A polynomial function of degree 1 is a linear function; its graph is a slant line. A polynomial function of degree 2 is a quadratic function; its graph is a parabola.

We now consider polynomial functions of higher degree. These include **cubic functions** (polynomials of degree 3) and **quartic functions** (polynomi-

Section 2.3 Exercises

In Exercises 1–6, describe how to transform the graph of an appropriate monomial function $f(x) = a_n x^n$ into the graph of the given polynomial function. Sketch the transformed graph by hand and support your answer with a grapher. Compute the location of the y-intercept as a check on the transformed graph.

1. $g(x) = 2(x - 3)^3$ **2.** $g(x) = -(x + 5)^3$

3. $g(x) = -\frac{1}{2}(x + 1)^3 + 2$ **4.** $g(x) = \frac{2}{3}(x - 3)^3 + 1$

5. $g(x) = -2(x + 2)^4 - 3$ **6.** $g(x) = 3(x - 1)^4 - 2$

In Exercises 7–10, state the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of the polynomial function by hand and support your answer with a grapher.

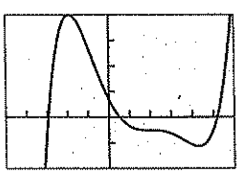
7. $f(x) = x(x - 3)^2$ **8.** $f(x) = -x^3(x - 2)$

9. $f(x) = (x - 2)^3(x + 1)^2$ **10.** $f(x) = 7(x - 3)^2(x + 5)^4$

In Exercises 11 and 12, graph the polynomial function, locate its extrema and zeros, and explain how it is related to the monomials from which it is built.

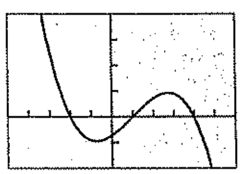
11. $f(x) = -x^4 + 2x$ **12.** $g(x) = 2x^4 - 5x^2$

In Exercises 13–16, match the polynomial function with its graph. Explain your choice.



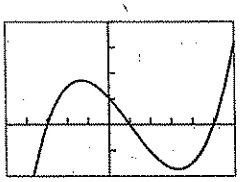
[–5, 6] by [–200, 400]

(a)



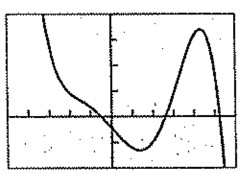
[–5, 6] by [–200, 400]

(b)



[–5, 6] by [–200, 400]

(c)



[–5, 6] by [–200, 400]

(d)

13. $f(x) = 7x^3 - 21x^2 - 91x + 104$

14. $f(x) = -9x^3 + 27x^2 + 54x - 73$

15. $f(x) = x^5 - 8x^4 + 9x^3 + 58x^2 - 164x + 69$

16. $f(x) = -x^5 + 3x^4 + 16x^3 - 2x^2 - 95x - 44$

In Exercises 17–20, graph the function pairs in the same series of viewing windows. Zoom out until the two graphs look nearly identical and state your final viewing window.

17. $f(x) = x^3 - 4x^2 - 5x - 3$ and $g(x) = x^3$

18. $f(x) = x^3 + 2x^2 - x + 5$ and $g(x) = x^3$

19. $f(x) = 2x^3 + 3x^2 - 6x - 15$ and $g(x) = 2x^3$

20. $f(x) = 3x^3 - 12x + 17$ and $g(x) = 3x^3$

In Exercises 21–28, graph the function in a viewing window that shows all of its extrema and x-intercepts.

21. $f(x) = (x - 1)(x + 2)(x + 3)$

22. $f(x) = (2x - 3)(4 - x)(x + 1)$

23. $f(x) = -x^3 + 4x^2 + 31x - 70$

24. $f(x) = x^3 - 2x^2 - 41x + 42$

25. $f(x) = (x - 2)^2(x + 1)(x - 3)$

26. $f(x) = (2x + 1)(x - 4)^3$

27. $f(x) = 2x^4 - 5x^3 - 17x^2 + 14x + 41$

28. $f(x) = -3x^4 - 5x^3 + 15x^2 - 5x + 19$

In Exercises 29–32, describe the end behavior of the polynomial function using $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

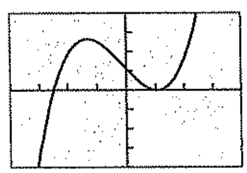
29. $f(x) = 3x^4 - 5x^2 + 3$

30. $f(x) = -x^3 + 7x^2 - 4x + 3$

31. $f(x) = 7x^2 - x^3 + 3x - 4$

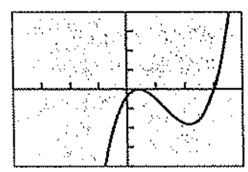
32. $f(x) = x^3 - x^4 + 3x^2 - 2x + 7$

In Exercises 33–36, match the polynomial function with its graph. Approximate all of the real zeros of the function.



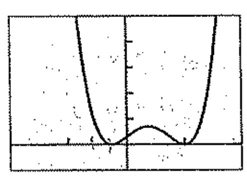
[–4, 4] by [–200, 200]

(a)



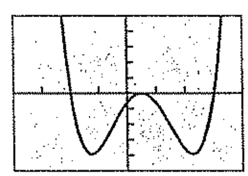
[–4, 4] by [–200, 200]

(b)



[–2, 2] by [–10, 50]

(c)



[–4, 4] by [–50, 50]

(d)

33. $f(x) = 20x^3 + 8x^2 - 83x + 55$

34. $f(x) = 35x^3 - 134x^2 + 93x - 18$

35. $f(x) = 44x^4 - 65x^3 + x^2 + 17x + 3$

36. $f(x) = 4x^4 - 8x^3 - 19x^2 + 23x - 6$

In Exercises 37–42, find the zeros of the function algebraically.

37. $f(x) = x^2 + 2x - 8$ **38.** $f(x) = 3x^2 + 4x - 4$

39. $f(x) = 9x^2 - 3x - 2$

40. $f(x) = x^3 - 25x$

41. $f(x) = 3x^3 - x^2 - 2x$

42. $f(x) = 5x^3 - 5x^2 - 10x$

In Exercises 43–48, graph the function in a viewing window that shows all of its x-intercepts and approximate all of its zeros.

43. $f(x) = 2x^3 + 3x^2 - 7x - 6$

44. $f(x) = -x^3 + 3x^2 + 7x - 2$

45. $f(x) = x^3 + 2x^2 - 4x - 7$

46. $f(x) = -x^4 - 3x^3 + 7x^2 + 2x + 8$

47. $f(x) = x^4 + 3x^3 - 9x^2 + 2x + 3$

48. $f(x) = 2x^5 - 11x^4 + 4x^3 + 47x^2 - 42x - 8$

In Exercises 49–52, find the zeros of the function algebraically or graphically.

49. $f(x) = x^3 - 36x$

50. $f(x) = x^3 + 2x^2 - 109x - 110$

51. $f(x) = x^3 - 7x^2 - 49x + 55$

52. $f(x) = x^3 - 4x^2 - 44x + 96$

In Exercises 53–56, using only algebra, find a cubic function with the given zeros. Support by graphing your answer.

53. 3, −4, 6

54. −2, 3, −5

55. $\sqrt{3}, -\sqrt{3}, 4$

56. $1, 1 + \sqrt{2}, 1 - \sqrt{2}$

57. Use cubic regression to fit a curve through the four points given in the table.

x	−3	−1	1	3
y	22	25	12	−5

58. Use cubic regression to fit a curve through the four points given in the table.

x	−2	1	4	7
y	2	5	9	26

59. Use quartic regression to fit a curve through the five points given in the table.

x	3	4	5	6	8
y	−7	−4	−11	8	3

60. Use quartic regression to fit a curve through the five points given in the table.

x	0	4	5	7	13
y	−21	−19	−12	8	3

61. **Analyzing Profit** Economists for Smith Brothers, Inc., find the company profit P by using the formula $P = R - C$, where R is the total revenue generated by the business and C is the total cost of operating the business.

(a) Using data from past years, the economists determined that $R(x) = 0.0125x^2 + 412x$ models total revenue, and $C(x) = 12{,}225 + 0.00135x^3$ models the total cost of doing business, where x is the number of customers patronizing the business. How many customers must Smith Bros. have to be profitable each year?

(b) How many customers must there be for Smith Bros. to realize an annual profit of $60,000?

62. **Stopping Distance** A state highway patrol safety division collected the data on stopping distances in Table 2.16.

(a) Draw a scatter plot of the data.

(b) Find the quadratic regression model.

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression model to predict the stopping distance for a vehicle traveling at 25 mph.

(e) Use the regression model to predict the speed of a car if the stopping distance is 300 ft.

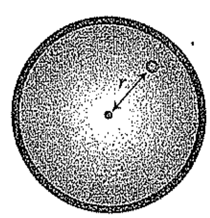
Table 2.16 Highway Safety Division

Speed (mph)	Stopping Distance (ft)
10	15.1
20	39.9
30	75.2
40	120.5
50	175.9

63. **Circulation of Blood** Research conducted at a national health research project shows that the speed at which a blood cell travels in an artery depends on its distance from the center of the artery. The function $v = 1.19 - 1.87r^2$ models the velocity (in centimeters per second) of a cell that is r centimeters from the center of an artery.

(a) Find a graph of v that reflects values of v appropriate for this problem. Record the viewing-window dimensions.

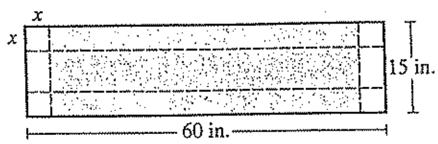
(b) If a blood cell is traveling at 0.975 cm/sec, estimate the distance the blood cell is from the center of the artery.



64. **Volume of a Box** Dixie Packaging Co. has contracted to manufacture a box with no top that is to be made by removing squares of width x from the corners of a 15-in. by 60-in. piece of cardboard.

(a) Show that the volume of the box is modeled by $V(x) = x(60 - 2x)(15 - 2x)$.

(b) Determine x so that the volume of the box is at least 450 in.^{3}



65. **Volume of a Box** Squares of width x are removed from a 10-cm by 25-cm piece of cardboard, and the resulting edges are folded up to form a box with no top. Determine all values of x so that the volume of the resulting box is at most 175 cm^3.

66. **Volume of a Box** The function $V = 2666x - 210x^2 + 4x^3$ represents the volume of a box that has been made by removing squares of width x from each corner of a rectangular sheet of material and then folding up the sides. What values are possible for x?

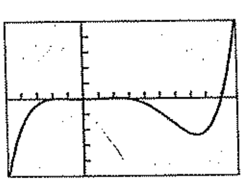
Explorations

In Exercises 67 and 68, two views of the function are given.

67. **Writing to Learn** Describe why each view of the function

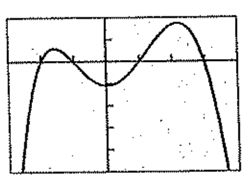
$$f(x) = x^5 - 10x^4 + 2x^3 + 64x^2 - 3x - 55,$$

by itself, may be considered inadequate.



[–5, 10] by [–7500, 7500]

(a)



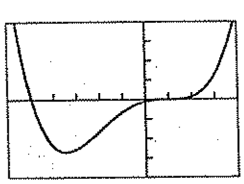
[–3, 4] by [–250, 100]

(b)

68. **Writing to Learn** Describe why each view of the function

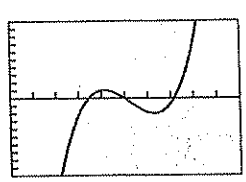
$$f(x) = 10x^4 + 19x^3 - 121x^2 + 143x - 51,$$

by itself, may be considered inadequate.



[–6, 4] by [–2000, 2000]

(a)



[0.5, 1.5] by [–1, 1]

(b)

In exercises 69–72, the function has hidden behavior when viewed in the window $[-10, 10]$ by $[-10, 10]$. Describe what behavior is hidden, and state the dimensions of a viewing widnow that reveals the hidden behavior.

69. $f(x) = 10x^3 - 40x^2 + 50x - 20$

70. $f(x) = 0.5(x^3 - 8x^2 + 12.99x - 5.94)$

71. $f(x) = 11x^3 - 10x^2 + 3x + 5$

72. $f(x) = 33x^3 - 100x^2 + 101x - 40$

Extending the Ideas

73. Graph the left side of the equation

$$3(x^3 - x) = a(x - b)^3 + c.$$

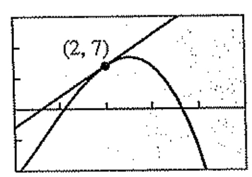
Then explain why there are no real numbers a, b, and c that make the equation true. (*Hint*: Use your knowledge of $y = x^3$ and transformations.)

74. Graph the left side of the equation

$$x^4 + 3x^3 - 2x - 3 = a(x - b)^4 + c.$$

Then explain why there are no real numbers a, b, and c that make the equation true.

75. **Looking Ahead to Calculus** The figure shows a graph of both $f(x) = -x^3 + 2x^2 + 9x - 11$ and the line L defined by $y = 5(x - 2) + 7$.



[0, 5] by [–10, 15]

(a) Confirm that the point $Q(2, 7)$ is a point of intersection of the two graphs.

(b) Zoom in at point Q to develop a visual understanding that $y = 5(x - 2) + 7$ is a *linear approximation* for $y = f(x)$ near $x = 2$.

(c) Recall that a line is *tangent* to a circle at a point P if it intersects the circle only at point P. View the two graphs in the window $[-5, 5]$ by $[-25, 25]$, and explain why that definition of tangent line is not valid for the graph of f.

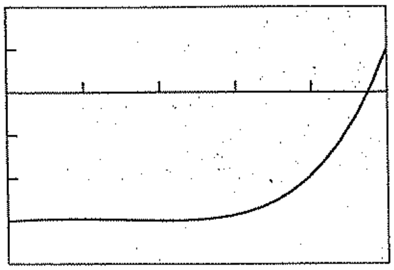
76. **Looking Ahead to Calculus** Consider the function $f(x) = x^n$ where n is an odd integer.

(a) Suppose that a is a positive number. Show that the slope of the line through the points $P(a, f(a))$ and $Q(-a, f(-a))$ is a^{n-1}.

(b) Let $x_0 = a^{1/(n-1)}$. Find an equation of the line through point $(x_0, f(x_0))$ with the slope a^{n-1}.

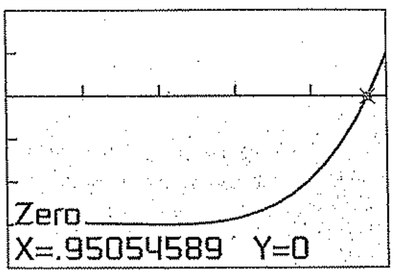
(c) Consider the special case $n = 3$ and $a = 3$. Show both the graph of f and the line from part b in the window $[-5, 5]$ by $[-30, 30]$.

77. **Derive an Algebraic Model of a Problem** Show that the distance x in the figure is a solution of the equation



[0, 1] by [–8, 4]

Figure 2.43 $y = 10x^5 - 3x^2 + x - 6$.



[0, 1] by [–8, 4]

Figure 2.44 An approximation for the irrational zero of $f(x) = 10x^5 - 3x^2 + x - 6$. (Example 9)

Solution We first prove that 1 is an upper bound and 0 is a lower bound on the real zeros of f. The function f has a positive leading coefficient, so we use synthetic division and the Upper and Lower Bounds Test:

$$\begin{array}{r|rrrrrrl} 1 & 10 & 0 & 0 & -3 & 1 & -6 & \\ & & 10 & 10 & 10 & 7 & 8 & \\ \hline & 10 & 10 & 10 & 7 & 8 & 2 & \text{Last line} \end{array}$$

$$\begin{array}{r|rrrrrrl} 0 & 10 & 0 & 0 & -3 & 1 & -6 & \\ & & 0 & 0 & 0 & 0 & 0 & \\ \hline & 10 & 0 & 0 & -3 & 1 & -6 & \text{Last line} \end{array}$$

Because the last line in the first division scheme consists of all positive numbers, 1 is an upper bound. Because the last line in the second division consists of numbers of alternating signs, 0 is a lower bound. All of the real zeros of f must therefore lie in the closed interval [0, 1]. So in Figure 2.43 we set our Xmin and Xmax accordingly.

Next we use the Rational Zeros Theorem.

Potential Rational Zeros:

$$\frac{\text{Factors of } -6}{\text{Factors of } 10}: \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 5, \pm 10}:$$

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}, \pm\frac{1}{10}, \pm\frac{3}{10}$$

We compare the x-intercepts of the graph in Figure 2.43 and our list of candidates, and decide f has no rational zeros. From Figure 2.43 we see that f changes sign on the interval [0.8, 1], so by the Intermediate Value Theorem must have a real zero on this interval. Because it is not rational we conclude that it is irrational. Figure 2.44 shows that this lone real zero of f is approximately 0.95.

Quick Review 2.4

In Exercises 1–4, rewrite the expression as a polynomial in standard form.

1. $\dfrac{x^3 - 4x^2 + 7x}{x}$
2. $\dfrac{2x^3 - 5x^2 - 6x}{2x}$
3. $\dfrac{x^4 - 3x^2 + 7x^5}{x^2}$
4. $\dfrac{6x^4 - 2x^3 + 7x^2}{3x^2}$

In Exercises 5–10, factor the polynomial into linear factors.

5. $x^3 - 4x$
6. $6x^2 - 54$
7. $4x^2 + 8x - 60$
8. $15x^3 - 22x^2 + 8x$
9. $x^3 + 2x^2 - x - 2$
10. $x^4 + x^3 - 9x^2 - 9x$

Section 2.4 Exercises

In Exercises 1–6, divide $f(x)$ by $d(x)$, and write a summary statement in polynomial form.

1. $f(x) = x^2 - 2x + 3$; $d(x) = x - 1$
2. $f(x) = x^3 - 1$; $d(x) = x + 1$
3. $f(x) = x^3 + 4x^2 + 7x - 9$; $d(x) = x + 3$
4. $f(x) = 4x^3 - 8x^2 + 2x - 1$; $d(x) = 2x + 1$
5. $f(x) = x^4 - 2x^3 + 3x^2 - 4x + 6$; $d(x) = x^2 + 2x - 1$
6. $f(x) = x^4 - 3x^3 + 6x^2 - 3x + 5$; $d(x) = x^2 + 1$

In Exercises 7–12, divide using synthetic division, and write a summary statement in fraction form.

7. $\dfrac{x^3 - 5x^2 + 3x - 2}{x + 1}$

8. $\dfrac{2x^4 - 5x^3 + 7x^2 - 3x + 1}{x - 3}$

9. $\dfrac{9x^3 + 7x^2 - 3x}{x - 10}$

10. $\dfrac{3x^4 + x^3 - 4x^2 + 9x - 3}{x + 5}$

11. $\dfrac{5x^4 - 3x + 1}{4 - x}$

12. $\dfrac{x^8 - 1}{x + 2}$

In Exercises 13–18, use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x - k$. Check by using synthetic division.

13. $f(x) = 2x^2 - 3x + 1; k = 2$

14. $f(x) = x^4 - 5; k = 1$

15. $f(x) = x^3 - x^2 + 2x - 1; k = -3$

16. $f(x) = x^3 - 3x + 4; k = -2$

17. $f(x) = 2x^3 - 3x^2 + 4x - 7; k = 2$

18. $f(x) = x^5 - 2x^4 + 3x^2 - 20x + 3; k = -1$

In Exercises 19–24, use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

19. $x - 1; x^3 - x^2 + x - 1$

20. $x - 3; x^3 - x^2 - x - 15$

21. $x - 2; x^3 + 3x - 4$

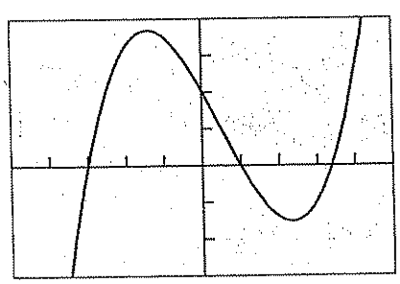
22. $x - 2; x^3 - 3x - 2$

23. $x + 2; 4x^3 + 9x^2 - 3x - 10$

24. $x + 1; 2x^{10} - x^9 + x^8 + x^7 + 2x^6 - 3$

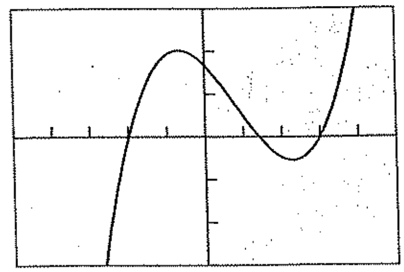
In Exercises 25 and 26, use the graph to guess possible linear factors of $f(x)$. Then completely factor $f(x)$ with the aid of synthetic division.

25. $f(x) = 5x^3 - 7x^2 - 49x + 51$



[–5, 5] by [–75, 100]

26. $f(x) = 5x^3 - 12x^2 - 23x + 42$



[–5, 5] by [–75, 75]

In Exercises 27–30, find the polynomial function with leading coefficient 2 that has the given degree and zeros.

27. Degree 3, with −2, 1, and 4 as zeros

28. Degree 3, with −1, 3, and −5 as zeros

29. Degree 3, with 2, $\frac{1}{2}$, and $\frac{3}{2}$ as zeros

30. Degree 4, with −3, −1, 0, and $\frac{5}{2}$ as zeros

In Exercises 31 and 32, using only algebraic methods, find the cubic function with the given table of values. Check with a grapher.

31.

x	−4	0	3	5
$f(x)$	0	180	0	0

32.

x	−2	−1	1	5
$f(x)$	0	24	0	0

In Exercises 33–36, use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros.

33. $f(x) = 6x^3 - 5x - 1$

34. $f(x) = 3x^3 - 7x^2 + 6x - 14$

35. $f(x) = 2x^3 - x^2 - 9x + 9$

36. $f(x) = 6x^4 - x^3 - 6x^2 - x - 12$

In Exercises 37–40, use synthetic division to prove that the number k is an upper bound for the real zeros of the function f.

37. $k = 3; f(x) = 2x^3 - 4x^2 + x - 2$

38. $k = 5; f(x) = 2x^3 - 5x^2 - 5x - 1$

39. $k = 2; f(x) = x^4 - x^3 + x^2 + x - 12$

40. $k = 3; f(x) = 4x^4 - 6x^3 - 7x^2 + 9x + 2$

In Exercises 41–44, use synthetic division to prove that the number k is a lower bound for the real zeros of the function f.

41. $k = -1; f(x) = 3x^3 - 4x^2 + x + 3$

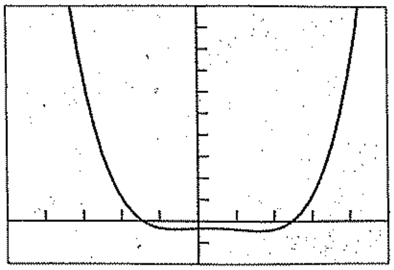
42. $k = -3; f(x) = x^3 + 2x^2 + 2x + 5$

43. $k = 0; f(x) = x^3 - 4x^2 + 7x - 2$

44. $k = -4; f(x) = 3x^3 - x^2 - 5x - 3$

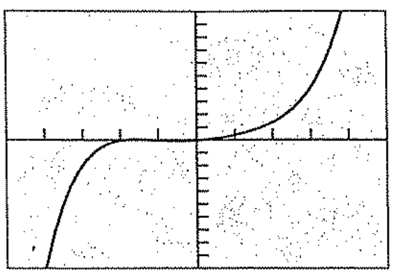
In Exercises 45–48, use the upper and lower bound tests to decide whether there is a real zero for the function outside the window shown.

45. $f(x) = 6x^4 - 11x^3 - 7x^2 + 8x - 34$



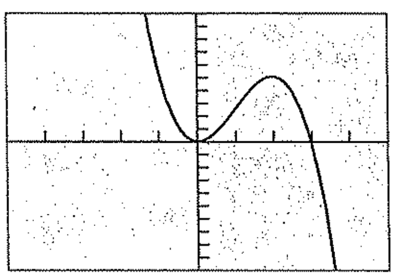
[–5, 5] by [–200, 1000]

46. $f(x) = x^5 - x^4 + 21x^2 + 19x - 3$



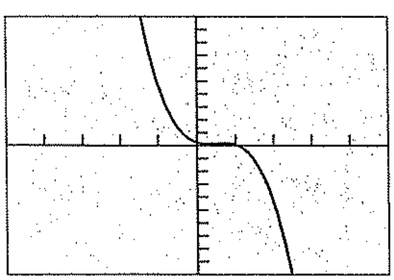
[–5, 5] by [–1000, 1000]

47. $f(x) = x^5 - 4x^4 - 129x^3 + 396x^2 - 8x + 3$



[–5, 5] by [–1000, 1000]

48. $f(x) = 2x^5 - 5x^4 - 141x^3 + 216x^2 - 91x + 25$



[–5, 5] by [–1000, 1000]

In Exercises 49–56, find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

49. $f(x) = 2x^3 - 3x^2 - 4x + 6$

50. $f(x) = x^3 + 3x^2 - 3x - 9$

51. $f(x) = x^3 + x^2 - 8x - 6$

52. $f(x) = x^3 - 6x^2 + 7x + 4$

53. $f(x) = x^4 - 3x^3 - 6x^2 + 6x + 8$

54. $f(x) = x^4 - x^3 - 7x^2 + 5x + 10$

55. $f(x) = 2x^4 - 7x^3 - 2x^2 - 7x - 4$

56. $f(x) = 3x^4 - 2x^3 + 3x^2 + x - 2$

57. **Setting Production Schedules** The Sunspot Small Appliance Co. determines that the supply function for their EverCurl hair dryer is $S(p) = 6 + 0.001p^3$ and that its demand function is $D(p) = 80 - 0.02p^2$, where p is the price. Determine the price for which the supply equals the demand and the number of hair dryers corresponding to this equilibrium price.

58. **Setting Production Schedules** The Pentkon Camera Co. determines that the supply and demand functions for their 35 mm – 70 mm zoom lens are $S(p) = 200 - p + 0.000007p^4$ and $D(p) = 1500 - 0.0004p^3$, where p is the price. Determine the price for which the supply equals the demand and the number of zoom lenses corresponding to this equilibrium price.

59. Find the remainder when $x^{40} - 3$ is divided by $x + 1$.

60. Find the remainder when $x^{63} - 17$ is divided by $x - 1$.

Explorations

61. Let $f(x) = x^4 + 2x^3 - 11x^2 - 13x + 38$

(a) Use the upper and lower bound tests to prove that all of the real zeros of f lie on the interval $[-5, 4]$.

(b) Find all of the rational zeros of f.

(c) Factor $f(x)$ using the rational zero(s) found in (b).

(d) Approximate all of the irrational zeros of f.

(e) Use synthetic division and the irrational zero(s) found in (d) to continue the factorization of $f(x)$ begun in (c).

62. **Retail Sales** The amount in billions of retail sales by Fuel Dealers for several years from 1985 to 1997 is given in Table 2.17. Let $x = 0$ stand for 1980, $x = 1$ for 1981, and so forth.

(a) Find a quartic regression model, and graph it together with a scatter plot of the data.

(b) Find a quadratic regression model, and graph it together with a scatter plot of the data.

(c) Use the quartic and quadratic regressions from (a) and (b) to get two estimates of the amount of retail sales for Fuel Dealers in 1999.

(d) **Writing to Learn** Give scenarios to justify each of the estimates in (c).

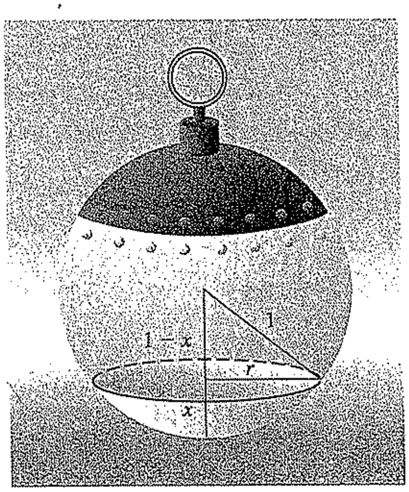
Table 2.17 Fuel Dealers Retail Sales

Year	Amount (billions)
1985	16.8
1990	15.6
1993	15.1
1994	16.0
1995	16.9
1996	19.0
1997	17.7

Source: U.S. Bureau of the Census, Current Busiess Reports, Statistical Abstracts of the United States, 1998.

63. **Archimedes' Principle** A spherical buoy has a radius of 1 m and a density one-fourth that of seawater. By Archimedes' Principle, the weight of the displaced water will equal the weight of the buoy.

- Let $x =$ the depth to which the buoy sinks.
- Let $d =$ the density of seawater.
- Let $r =$ the radius of the circle formed where buoy, air, and water meet. See the figure below.



Notice that $0 < x < 1$ and that

$$(1 - x)^2 + r^2 = 1,$$
$$r^2 = 1 - (1 - x)^2$$
$$= 2x - x^2.$$

(a) Verify that the volume of the buoy is $4\pi/3$.

(b) Use your result from (a) to establish the weight of the buoy as $\pi d/3$.

(c) Prove the weight of the displaced water is $\pi d \cdot x(3r^2 + x^2)/6$.

(d) Approximate the depth to which the buoy will sink.

64. **Archimedes' Principle** Using the scenario of Exercise 63, find the depth to which the buoy will sink if its density is one-fifth that of seawater.

65. **Biological Research** Stephanie, a biologist who does research for the poultry industry, models the population P of wild turkeys, t days after being left to reproduce, with the function

$$P(t) = -0.00001t^3 + 0.002t^2 + 1.5t + 100.$$

(a) Graph the function $y = P(t)$ for appropriate values of t.

(b) Find what the maximum turkey population is and when it occurs.

(c) Assuming that this model continues to be accurate, when will this turkey population become extinct?

(d) **Writing to Learn** Create a scenario that could explain the growth exhibited by this turkey population.

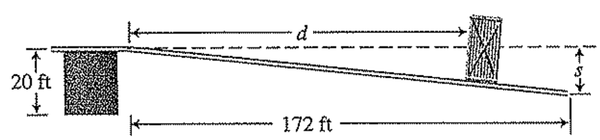
66. **Architectural Engineering** Dave, an engineer at the Trumbauer Group, Inc., an architectural firm, completes structural specifications for a 172-ft-long steel beam, anchored at one end to a piling 20 ft above the ground. He knows that when a 200-lb object is placed d feet from the anchored end, the beam bends s feet where

$$s = (3 \times 10^{-7})d^2(550 - d).$$

(a) What is the independent variable in this polynomial function?

(b) What are the dimensions of a viewing window that shows a graph for the values that make sense in this problem situation?

(c) How far is the 200-lb object from the anchored end if the vertical deflection is 1.25 ft?



Extending the Ideas

67. **Writing to Learn** Graph each side of the Example 3 summary equation:

$$f(x) = \frac{2x^3 - 3x^2 - 5x - 12}{x - 3} \quad \text{and}$$

$$g(x) = 2x^2 + 3x + 4, \quad x \neq 3$$

How are these functions related? Include a discussion of the domain and continuity of each function.

68. **Writing to Learn** Graph each side of the Example 4 summary equation:

$$f(x) = \frac{x^4 - 8x^3 + 11x - 6}{x + 3} \quad \text{and}$$

$$g(x) = x^3 - 11x^2 + 33x - 88 + \frac{258}{x + 3}$$

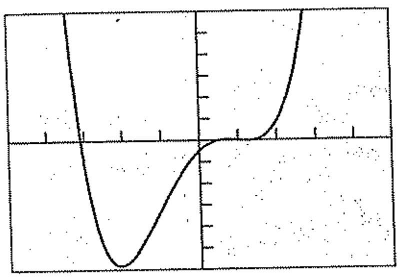
How are these functions related? Include a discussion of the domain and continuity of each function.

69. **Writing to Learn** Explain how to carry out the following division using synthetic division. Work through the steps with complete explanations.

$$\frac{4x^3 - 5x^2 + 3x + 1}{2x - 1}$$

70. **Writing to Learn** The figure shows a graph of $f(x) = x^4 + 0.1x^3 - 6.5x^2 + 7.9x - 2.4$. Explain how to use a grapher to justify the statement.

$$f(x) = x^4 + 0.1x^3 - 6.5x^2 + 7.9x - 2.4$$
$$\approx (x + 3.10)(x - 0.5)(x - 1.13)(x - 1.37)$$



[–5, 5] by [–30, 30]

71. (a) **Writing to Learn** Write a paragraph that describes how the zeros of $f(x) = (1/3)x^3 + x^2 + 2x - 3$ are related to the zeros of $g(x) = x^3 + 3x^2 + 6x - 9$. In what ways does this example illustrate how the Rational Zeros Theorem can be applied to find the zeros of a polynomial with *rational* number coefficients?

(b) Find the rational zeros of $f(x) = x^3 - \frac{7}{6}x^2 - \frac{20}{3}x + \frac{7}{2}$.

(c) Find the rational zeros of $f(x) = x^3 - \frac{5}{2}x^2 - \frac{37}{12}x + \frac{5}{2}$.

72. Use the Rational Zeros Theorem to prove $\sqrt{2}$ is irrational.

73. **Group Activity** *Work in groups of three.* Graph $f(x) = x^4 + x^3 - 8x^2 - 2x + 7$.

(a) Use grapher methods to find approximate real number zeros.

(b) Identify a list of four linear factors whose product could be called an *approximate factorization of* $f(x)$.

(c) Discuss what graphical and numerical methods you could use to show that the factorization from (b) is reasonable.

74. A classic theorem, **Descartes' Rule of Signs**, tells us about the number of positive and negative real zeros of a polynomial function, by looking at the polynomial's variations in sign. A *variation in sign* occurs when consecutive coefficients (in standard form) have opposite signs.

If $f(x) = a_n x^n + \cdots + a_0$ is a polynomial of degree n, then

- The number of positive real zeros of f is equal to the number of variations in sign of $f(x)$, or that number less some even number.
- The number of negative real zeros of f is equal to the number of variations in sign of $f(-x)$, or that number less some even number.

Use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of the function.

(a) $f(x) = x^3 + x^2 - x + 1$

(b) $f(x) = x^3 + x^2 + x + 1$

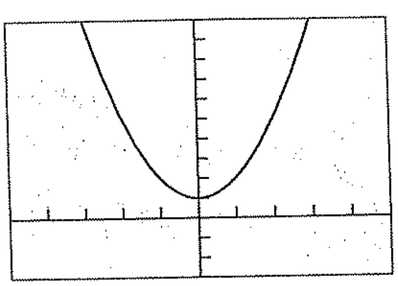
(c) $f(x) = 2x^3 + x - 3$

(d) $g(x) = 5x^4 + x^2 - 3x - 2$

2.5 Complex Numbers

Complex Numbers • Operations with Complex Numbers • Complex Conjugates and Division • Complex Solutions of Quadratic Equations • Complex Plane

Complex Numbers



[–5, 5] by [–3, 10]

Figure 2.45 The graph of $f(x) = x^2 + 1$ has no x-intercepts.

Figure 2.45 shows that the function $f(x) = x^2 + 1$ has no real zeros, so $x^2 + 1 = 0$ has no real-number solutions. To remedy this situation, mathematicians in the 17th century extended the definition of $\sqrt{a}$ to include negative real numbers a. First the number $i = \sqrt{-1}$ is defined as a solution of the equation $i^2 + 1 = 0$ and is the **imaginary unit**. Then for any negative real number a, $\sqrt{a} = \sqrt{|a|} \cdot i$.

The extended system of numbers, called the *complex numbers*, consists of all real numbers and sums of real numbers and real number multiples of i. The following are all examples of complex numbers:

$$-6, \quad 5i, \quad \sqrt{5}, \quad -7i, \quad \frac{5}{2}i \quad \frac{2}{3}, \quad -2 + 3i, \quad 5 - 3i, \quad \frac{1}{3} + \frac{4}{5}i$$

Section 2.7 Exercises

In Exercises 1–8, which of the following are rational functions? For those that are rational functions, state the domain. For those that are not, explain why not.

1. $f(x) = \dfrac{2x^2 - 3x + 4}{x^3 + x + 1}$

2. $f(x) = \dfrac{3x - 1}{x^2 + \sqrt{x - 1}}$

3. $f(x) = \dfrac{x^2 - 3\sqrt{x} + 1}{2x - 3}$

4. $f(x) = \dfrac{1}{3x + 4}$

5. $f(x) = \dfrac{1}{x} - 4$

6. $f(x) = 2 - \dfrac{1}{x}$

7. $f(x) = 4 + \dfrac{1}{x + 1}$

8. $f(x) = -2 + \dfrac{2}{x + 3}$

In Exercises 9–16, describe how the graph of the given function can be obtained by transforming the graph of the reciprocal function $f(x) = 1/x$. Identify the horizontal and vertical asymptotes.

9. $f(x) = \dfrac{1}{x - 3}$

10. $f(x) = \dfrac{1}{x + 3}$

11. $f(x) = -\dfrac{2}{x + 5}$

12. $f(x) = \dfrac{2}{x + 2}$

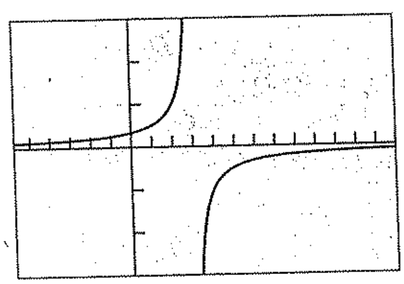
13. $f(x) = \dfrac{2x - 1}{x + 3}$

14. $f(x) = \dfrac{3x - 2}{x - 1}$

15. $f(x) = \dfrac{5 - 2x}{x + 4}$

16. $f(x) = \dfrac{4 - 3x}{x - 5}$

In Exercises 17–20, evaluate the limit based on the graph of f shown.



[–5.8, 13] by [–3, 3]

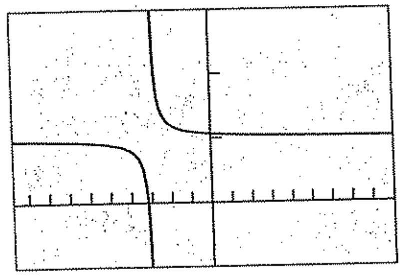
17. $\lim\limits_{x \to 3^-} f(x)$

18. $\lim\limits_{x \to 3^+} f(x)$

19. $\lim\limits_{x \to \infty} f(x)$

20. $\lim\limits_{x \to -\infty} f(x)$

In Exercises 21–24, evaluate the limit based on the graph of f shown.



[–9.8, 9] by [–5, 15]

21. $\lim\limits_{x \to -3^+} f(x)$

22. $\lim\limits_{x \to -3^-} f(x)$

23. $\lim\limits_{x \to -\infty} f(x)$

24. $\lim\limits_{x \to \infty} f(x)$

In Exercises 25–36, find the asymptotes and intercepts of the function, and graph the function.

25. $f(x) = \dfrac{2}{x - 3}$

26. $f(x) = \dfrac{-3}{x + 2}$

27. $g(x) = \dfrac{x - 2}{x^2 - 2x - 3}$

28. $g(x) = \dfrac{x + 2}{x^2 + 2x - 3}$

29. $h(x) = \dfrac{2}{x^3 - x}$

30. $h(x) = \dfrac{3}{x^3 - 4x}$

31. $k(x) = \dfrac{x - 1}{x^2 + 3}$

32. $k(x) = \dfrac{x + 3}{x^2 + 1}$

33. $f(x) = \dfrac{2x^2 + x - 2}{x^2 - 1}$

34. $g(x) = \dfrac{-3x^2 + x + 12}{x^2 - 4}$

35. $f(x) = \dfrac{x^2 - 2x + 3}{x + 2}$

36. $g(x) = \dfrac{x^2 - 3x - 7}{x + 3}$

In Exercises 37–42, match the rational function with its graph. Identify the viewing window and the scale used on each axis.

37. $f(x) = \dfrac{1}{x - 4}$

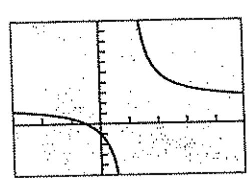
38. $f(x) = -\dfrac{1}{x + 3}$

39. $f(x) = 2 + \dfrac{3}{x - 1}$

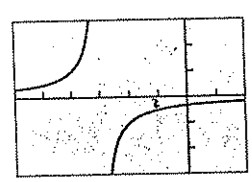
40. $f(x) = 1 + \dfrac{1}{x + 3}$

41. $f(x) = -1 + \dfrac{1}{4 - x}$

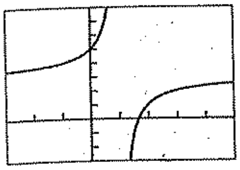
42. $f(x) = 3 - \dfrac{2}{x - 1}$



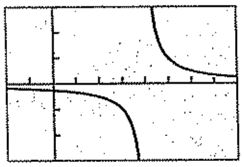
(a)



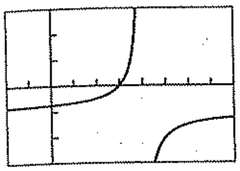
(b)



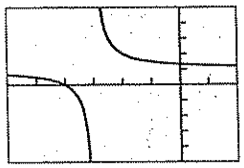
(c)



(d)



(e)



(f)

In Exercises 43–48, solve the equation algebraically. Check for extraneous solutions. Support your answer graphically.

43. $\dfrac{3x}{x+5} + \dfrac{1}{x-2} = \dfrac{7}{x^2 + 3x - 10}$

44. $\dfrac{4x}{x+4} + \dfrac{3}{x-1} = \dfrac{15}{x^2 + 3x - 4}$

45. $\dfrac{x-3}{x} - \dfrac{3}{x+1} + \dfrac{3}{x^2 + x} = 0$

46. $\dfrac{x+2}{x} - \dfrac{4}{x-1} + \dfrac{2}{x^2 - x} = 0$

47. $\dfrac{3}{x+2} + \dfrac{6}{x^2 + 2x} = \dfrac{3-x}{x}$

48. $\dfrac{x+3}{x} - \dfrac{2}{x+3} = \dfrac{6}{x^2 + 3x}$

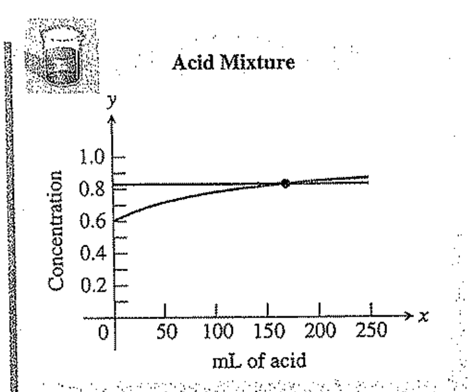
49. **Acid Mixture** Suppose that x mL of pure acid are added to 125 mL of a 60% acid solution. How many mL of pure acid must be added to obtain a solution of at least 83% acid?

(a) Explain why the concentration $C(x)$ of the new mixture is

$$C(x) = \frac{x + 0.6(125)}{x + 125}.$$

(b) Suppose the viewing window in the figure is used to find a solution to the problem. What is the equation of the horizontal line?

(c) Write and solve an inequality that answers the question of this problem.



50. **Acid Mixture** Suppose that x mL of pure acid are added to 100 mL of a 35% acid solution.

(a) Express the concentration $C(x)$ of the new mixture as a function of x.

(b) Use a graph to determine how much pure acid should be added to the 35% solution to produce a new solution that is less than 75% acid.

(c) Solve (b) algebraically.

51. **Breaking Even** Mid Town Sports Apparel, Inc., has found that it needs to sell golf hats for \$2.75 each in order to be competitive. It costs \$2.12 to produce each hat, and it has weekly overhead costs of \$3000.

(a) Let x be the number of hats produced each week. Express the average cost (including overhead costs) of producing one hat as a function of x.

(b) Solve algebraically to find the number of golf hats that must be sold each week to make a profit. Support your answer graphically.

(c) How many golf hats must be sold to make a profit of \$1000 in 1 week?

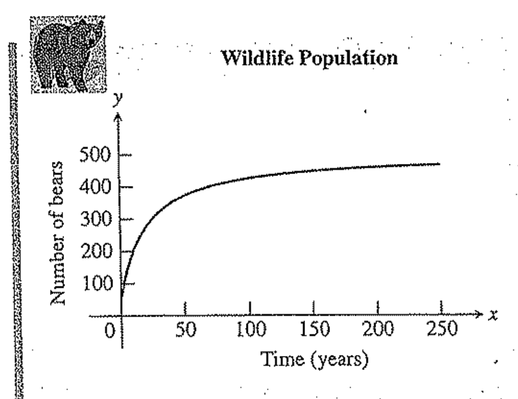
52. **Bear Population** The number of bears at any time t (in years) in a federal game reserve is given by

$$P(t) = \frac{500 + 250t}{10 + 0.5t}.$$

(a) Find the population of bears when the value of t is 10, 40, and 100.

(b) Does the graph of the bear population have a horizontal asymptote? If so, what is it? If not, why not?

(c) According to this model, what is the largest the bear population can become?



Explorations

53. **Group Activity** *Work in groups of two.* Compare the functions

$$f(x) = \frac{x^2 - 9}{x - 3} \quad \text{and} \quad g(x) = x + 3.$$

(a) Are the domains equal?

(b) Does f have a vertical asymptote? Explain.

(c) Explain why the graphs appear to be identical.

(d) Are the functions identical?

54. **Group Activity** Explain why the functions are identical or not. Include the graphs and a comparison of the functions' asymptotes, intercepts, and domain.

(a) $f(x) = \dfrac{x^2 + x - 2}{x - 1}$ and $g(x) = x + 2$

(b) $f(x) = \dfrac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$

(c) $f(x) = \dfrac{x^2 - 1}{x^3 - x^2 - x + 1}$ and $g(x) = \dfrac{1}{x - 1}$

(d) $f(x) = \dfrac{x - 1}{x^2 + x - 2}$ and $g(x) = \dfrac{1}{x + 2}$

55. **Boyle's Law** This ideal gas law states that the volume of an enclosed gas at a fixed temperature varies inversely as the pressure.

(a) **Writing to Learn** Explain why Boyle's Law yields both a rational function model and a power function model.

(b) Which power functions are also rational functions?

(c) If the pressure of a 2.59-L sample of nitrogen gas at 291°K is 0.866 atm, what would the volume be at a pressure of 0.532 atm if the temperature does not change?

56. **Light Intensity** Aileen and Malachy gathered the data in Table 2.22 using a 75-watt light bulb and a Calculator-Based Laboratory™ (CBL™) with a light-intensity probe.

(a) Draw a scatter plot of the data in Table 2.22.

(b) Find an equation for the data assuming it has the form $f(x) = k/x^2$ for some constant k. Explain your method for choosing k.

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression model to predict the light intensity at distances of 2.2 m and 4.4 m.

Table 2.22 Light Intensity Data for a 75-W Light Bulb

Distance (m)	Intensity (W/m^2)
1.0	6.09
1.5	2.51
2.0	1.56
2.5	1.08
3.0	0.74

Extending the Ideas

In Exercises 57–60, find the asymptotes and intercepts of the function, and graph the function. Overlay a graph of the end behavior asymptote.

57. $f(x) = \dfrac{x^3 - 2x^2 + x - 1}{2x - 1}$

58. $g(x) = \dfrac{2x^3 - 2x^2 - x + 5}{x - 2}$

59. $f(x) = \dfrac{2x^4 - x^3 - 16x^2 + 17x - 6}{2x - 5}$

60. $g(x) = \dfrac{2x^5 - 3x^3 + 2x - 4}{x - 1}$

In Exercises 61–64, graph the function. Express the function as a piecewise-defined function without absolute value, and use the result to confirm the graph's asymptotes and intercepts algebraically.

61. $h(x) = \dfrac{2x - 3}{|x| + 2}$

62. $h(x) = \dfrac{3x + 5}{|x| + 3}$

63. $f(x) = \dfrac{5 - 3x}{|x| + 4}$

64. $f(x) = \dfrac{2 - 2x}{|x| + 1}$

65. Describe how the graph of

$$f(x) = \frac{ax + b}{cx + d}$$

can be obtained from the graph of $y = 1/x$. (*Hint:* Use long division.)

66. **Writing to Learn** Let $f(x) = 1/(x - (1/x))$ and $g(x) = (x^3 + x^2 - x)/(x^3 - x)$. Does $f = g$? Support your answer by making a comparative analysis of all of the features of f and g, including asymptotes, intercepts, and domain.

Quick Review 2.8

In Exercises 1–4, use limits to state the end behavior of the function.

1. $f(x) = 2x^3 + 3x^2 - 2x + 1$
2. $f(x) = -3x^4 - 3x^3 + x^2 - 1$
3. $g(x) = \dfrac{x^3 - 2x^2 + 1}{x - 2}$
4. $g(x) = \dfrac{2x^2 - 3x + 1}{x + 1}$

In Exercises 5–8, combine the fractions and reduce your answer to lowest terms.

5. $x^2 + \dfrac{5}{x}$
6. $x^2 - \dfrac{3}{x}$
7. $\dfrac{x}{2x + 1} - \dfrac{2}{x - 3}$
8. $\dfrac{x}{x - 1} + \dfrac{x + 1}{3x - 4}$

In Exercises 9 and 10, **(a)** list all the possible rational zeros of the polynomial and **(b)** factor the polynomial completely.

9. $2x^3 + x^2 - 4x - 3$
10. $3x^3 - x^2 - 10x + 8$

Section 2.8 Exercises

In Exercises 1–6, determine the x values that cause the polynomial function to be **(a)** zero, **(b)** positive, and **(c)** negative.

1. $f(x) = (x + 2)(x + 1)(x - 5)$
2. $f(x) = (x - 7)(3x + 1)(x + 4)$
3. $f(x) = (x + 7)(x + 4)(x - 6)^2$
4. $f(x) = (5x + 3)(x^2 + 6)(x - 1)$
5. $f(x) = (2x^2 + 5)(x - 8)^2(x + 1)^3$
6. $f(x) = (x + 2)^3(4x^2 + 1)(x - 9)^4$

In Exercises 7–12, complete the factoring if needed, and solve the polynomial inequality using a sign chart. Support graphically.

7. $(x + 1)(x - 3)^2 > 0$
8. $(2x + 1)(x - 2)(3x - 4) \le 0$
9. $(x + 1)(x^2 - 3x + 2) < 0$
10. $(2x - 7)(x^2 - 4x + 4) > 0$
11. $2x^3 - 3x^2 - 11x + 6 \ge 0$
12. $x^3 - 4x^2 + x + 6 \le 0$

In Exercises 13–20, solve the polynomial inequality graphically.

13. $x^3 - x^2 - 2x \ge 0$
14. $2x^3 - 5x^2 + 3x < 0$
15. $2x^3 - 5x^2 - x + 6 > 0$
16. $x^3 - 4x^2 - x + 4 \le 0$
17. $3x^3 - 2x^2 - x + 6 \ge 0$
18. $-x^3 - 3x^2 - 9x + 4 < 0$
19. $2x^4 - 3x^3 - 6x^2 + 5x + 6 < 0$
20. $3x^4 - 5x^3 - 12x^2 + 12x + 16 \ge 0$

In Exercises 21–28, determine the x values that cause the function to be **(a)** zero, **(b)** undefined, **(c)** positive, and **(d)** negative.

21. $f(x) = \dfrac{x - 1}{(2x + 3)(x - 4)}$
22. $f(x) = \dfrac{(2x - 7)(x + 1)}{x + 5}$
23. $f(x) = x\sqrt{x + 3}$
24. $f(x) = x^2|2x + 9|$
25. $f(x) = \dfrac{\sqrt{x + 5}}{(2x + 1)(x - 1)}$
26. $f(x) = \dfrac{x - 1}{(x - 4)\sqrt{x + 2}}$
27. $f(x) = \dfrac{(2x + 5)\sqrt{x - 3}}{(x - 4)^2}$
28. $f(x) = \dfrac{3x - 1}{(x + 3)\sqrt{x - 5}}$

In Exercises 29–40, solve the inequality using a sign chart. Support graphically.

29. $\dfrac{x - 1}{x^2 - 4} < 0$
30. $\dfrac{x + 2}{x^2 - 9} < 0$
31. $\dfrac{x^2 - 1}{x^2 + 1} \le 0$
32. $\dfrac{x^2 - 4}{x^2 + 4} > 0$
33. $\dfrac{x^2 + x - 12}{x^2 - 4x + 4} > 0$
34. $\dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$
35. $\dfrac{x^3 - x}{x^2 + 1} \ge 0$
36. $\dfrac{x^3 - 4x}{x^2 + 2} \le 0$
37. $x|x - 2| > 0$
38. $\dfrac{x - 3}{|x + 2|} < 0$
39. $(2x - 1)\sqrt{x + 4} < 0$
40. $(3x - 4)\sqrt{2x + 1} \ge 0$

In Exercises 41–50, solve the inequality.

41. $\dfrac{x^3(x - 2)}{(x + 3)^2} < 0$
42. $\dfrac{(x - 5)^4}{x(x + 3)} \ge 0$
43. $x^2 - \dfrac{2}{x} > 0$
44. $x^2 + \dfrac{4}{x} \ge 0$
45. $\dfrac{1}{x + 1} + \dfrac{1}{x - 3} \le 0$
46. $\dfrac{1}{x + 2} - \dfrac{2}{x - 1} > 0$
47. $(x + 3)|x - 1| \ge 0$
48. $(3x + 5)^2|x - 2| < 0$
49. $\dfrac{(x - 5)|x - 2|}{\sqrt{2x - 3}} \ge 0$
50. $\dfrac{x^2(x - 4)^3}{\sqrt{x + 1}} < 0$

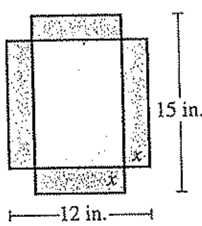
51. **Writing to Learn** Write a paragraph that explains two ways to solve the inequality $3(x - 1) + 2 \le 5x + 6$.

52. **Company Wages** Pederson Electric Co. charges \$25 per service call plus \$18 per hour for home repair work. How long did an electrician work if the charge was less than \$100? Assume the electrician rounds off the time to the nearest quarter hour.

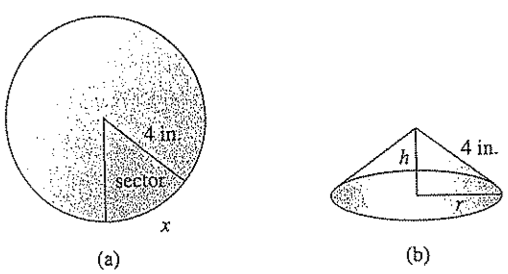
53. **Connecting Algebra and Geometry** Consider the collection of all rectangles that have lengths 2 in. less than twice their widths. Find the possible widths (in inches) of these rectangles if their perimeters are less than 200 in.

54. **Planning for Profit** The Grovenor Candy Co. finds that the cost of making a certain candy bar is \$0.13 per bar. Fixed costs amount to \$2000 per week. If each bar sells for \$0.35, find the minimum number of candy bars sold per week that will earn the company a profit.

55. **Designing a Cardboard Box** Picaro's Packaging Plant wishes to design boxes with a volume of *not more than* 100 in.^{3} Squares are to be cut from the corners of a 12-in. by 15-in. piece of cardboard (see figure), with the flaps folded up to make an open box. What size squares should be cut from the cardboard?



56. **Cone Problem** Beginning with a circular piece of paper with a 4-inch radius, as shown in (a), cut out a sector with an arc of length x. Join the two radial edges of the remaining portion of the paper to form a cone with radius r and height h, as shown in (b). What length of arc will produce a cone with a volume greater than 21 in.^{3}?



57. **Resistors** The total electrical resistance R of two resistors connected in parallel with resistances R_1 and R_2 is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

One resistor has a resistance of 2.3 ohms. Let x be the resistance of the second resistor.

(a) Express the total resistance R as a function of x.

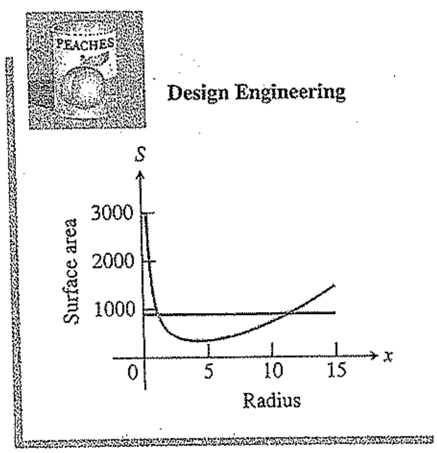
(b) Find the resistance in the second resistor if the total resistance of the pair is at least 1.7 ohms.

58. **Design a Juice Can** Flannery Cannery packs peaches in 0.5-L cylindrical cans.

(a) Express the surface area S of the can as a function of the radius x (in cm).

(b) Find the dimensions of the can if the surface is less than 900 cm^2.

(c) Find the least possible surface area of the can.



59. **Per Capita Income** The U.S. average per capita income for several years from 1990 to 1997 is given in Table 2.23. Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth.

(a) Find the linear regression model for the data and superimpose its graph on a scatter plot of the data.

(b) Use the model to predict when the per capita income exceeds \$30,000.

Table 2.23 Per Capita Income

Year	Amount (dollars)
1990	19,188
1995	23,359
1996	24,436
1997	25,598

Source: U.S. Bureau of the Census, Survey of Current Business, Statistical Abstract of the United States, 1998

60. Annual Housing Cost The average annual expenditure for housing for several years from 1989 to 1995 is given in Table 2.24. Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth.

(a) Find the linear regression model for the data and superimpose its graph on a scatter plot of the data.

(b) Use the model to predict when the average annual expenditure for housing exceeds $12,000.

Table 2.24 Average Annual Housing Cost

Year	Amount (dollars)
1989	8434
1990	8703
1991	9252
1992	9477
1993	9636
1994	10,106
1995	10,465

Source: U.S. Bureau of the Census, Consumer Expenditures, Statistical Abstract of the United States, 1998

Explorations

In Exercises 61 and 62, find the vertical asymptotes and intercepts of the rational function. Then use a sign chart and a table of values to graph the function by hand. Support your result using a grapher. (Hint: You may need to graph the function in more than one window to see different parts of the overall graph.)

61. $f(x) = \dfrac{(x-1)(x+2)^2}{(x-3)(x+1)}$ **62.** $g(x) = \dfrac{(x-3)^4}{x^2+4x}$

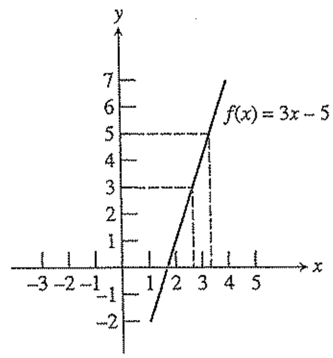
Extending the Ideas

63. Group Activity Looking Ahead to Calculus Let $f(x) = 3x - 5$.

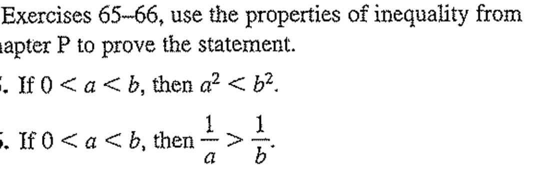
(a) Assume x is in the interval defined by $|x - 3| < 1/3$. Give a convincing argument that $|f(x) - 4| < 1$.

(b) **Writing to Learn** Explain how (a) is modeled by the figure at the top of the next column.

(c) Show how the algebra used in (a) can be modified to show that if $|x - 3| < 0.01$, then $|f(x) - 4| < 0.03$. How would the figure below change to reflect these inequalities?



64. Writing to Learn Boolean Operators The Test menu of many graphers contains inequality symbols that can be used to construct inequality statements, as shown in (a). An answer of 1 indicates the statement is true, and 0 indicates the statement is false. In (b), the graph of $Y_1 = x^2 - 4 \geq 0$ is shown using Dot mode and the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$. Experiment with the Test menu, and then write a paragraph explaining how to interpret the graph in (b).



[−4.7, 4.7] by [−3.1, 3.1]

(a) (b)

In Exercises 65–66, use the properties of inequality from Chapter P to prove the statement.

65. If $0 < a < b$, then $a^2 < b^2$.

66. If $0 < a < b$, then $\dfrac{1}{a} > \dfrac{1}{b}$.

Chapter 2 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1 and 2, write an equation for the linear function f satisfying the given conditions. Graph $y = f(x)$.

1. $f(-3) = -2$ and $f(4) = -9$
2. $f(-3) = 6$ and $f(1) = -2$

In Exercises 3 and 4, describe how to transform the graph of $f(x) = x^2$ into the graph of the given function. Sketch the graph by hand and support your answer with a grapher.

3. $f(x) = 3(x - 2)^2 + 4$
4. $g(x) = -(x + 3)^2 + 1$

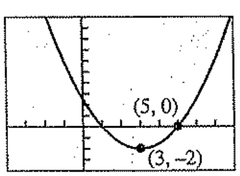
In Exercises 5–8, find the vertex and axis of the graph of the function. Support your answer graphically.

5. $f(x) = -2(x + 3)^2 + 5$
6. $g(x) = 4(x - 5)^2 - 7$
7. $f(x) = -2x^2 - 16x - 31$
8. $g(x) = 3x^2 - 6x + 2$

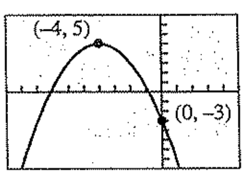
In Exercises 9 and 10, write an equation for the quadratic function whose graph contains the given vertex and point.

9. Vertex $(-2, -3)$, point $(1, 2)$
10. Vertex $(-1, 1)$, point $(3, -2)$

In Exercises 11 and 12, write an equation for the quadratic function with graph shown, given one of the labeled points is the vertex of the parabola.

11. 

[–4, 8] by [–4, 10]

12. 

[–10, 5] by [–8, 8]

In Exercises 13–16, graph the function in a viewing window that shows all of its extrema and x-intercepts.

13. $f(x) = x^2 + 3x - 40$
14. $f(x) = -8x^2 + 16x - 19$
15. $f(x) = x^3 + x^2 + x + 5$
16. $f(x) = x^3 - x^2 - 20x - 2$

In Exercises 17 and 18, write the statement as a power function equation.

17. The surface area S of a sphere varies directly as the square of the radius r.
18. The force of gravity F acting on an object is inversely proportional to the square of the distance d from the object to the center of the earth.

In Exercises 19 and 20, write a sentence that expresses the relationship in the formula, using the language of variation or proportion.

19. $F = kx$, where F is the force it takes to stretch a spring x units from its unstressed length and k is the spring's force constant.
20. $A = \pi \cdot r^2$, where A and r are the area and radius of a circle and π is the usual mathematical constant.

In Exercises 21–24, state the values of the constants k and a for the function $f(x) = k \cdot x^a$. Describe the portion of the curve that lies in Quadrant I or IV. Determine whether f is even, odd, or undefined for $x < 0$. Describe the rest of the curve if any. Graph the function to see whether it matches the description.

21. $f(x) = 4x^{1/3}$
22. $f(x) = -2x^{3/4}$
23. $f(x) = -2x^{-3}$
24. $f(x) = (2/3)x^{-4}$

In Exercises 25–28, divide $f(x)$ by $d(x)$, and write a summary statement in polynomial form.

25. $f(x) = 2x^3 - 7x^2 + 4x - 5$; $d(x) = x - 3$
26. $f(x) = x^4 + 3x^3 + x^2 - 3x + 3$; $d(x) = x + 2$
27. $f(x) = 2x^4 - 3x^3 + 9x^2 - 14x + 7$; $d(x) = x^2 + 4$
28. $f(x) = 3x^4 - 5x^3 - 2x^2 + 3x - 6$; $d(x) = 3x + 1$

In Exercises 29 and 30, use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x - k$. Check by using synthetic division.

29. $f(x) = 3x^3 - 2x^2 + x - 5$; $k = -2$
30. $f(x) = -x^2 + 4x - 5$; $k = 3$

In Exercises 31 and 32, use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

31. $x - 2$; $x^3 - 4x^2 + 8x - 8$
32. $x + 3$; $x^3 + 2x^2 - 4x - 2$

In Exercises 33 and 34, use synthetic division to prove that the number k is an upper bound for the real zeros of the function f.

33. $k = 5$; $f(x) = x^3 - 5x^2 + 3x + 4$
34. $k = 4$; $f(x) = 4x^4 - 16x^3 + 8x^2 + 16x - 12$

In Exercises 35 and 36, use synthetic division to prove that the number k is a lower bound for the real zeros of the function f.

35. $k = -3$; $f(x) = 4x^4 + 4x^3 - 15x^2 - 17x - 2$
36. $k = -3$; $f(x) = 2x^3 + 6x^2 + x - 6$

In Exercises 37 and 38, use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros.

37. $f(x) = 2x^4 - x^3 - 4x^2 - x - 6$
38. $f(x) = 6x^3 - 20x^2 + 11x + 7$

In Exercises 39–46, perform the indicated operation, and write the result in the standard form $a + bi$.

39. $(3 - 2i) + (-2 + 5i)$
40. $(5 - 7i) - (3 - 2i)$
41. $(1 + 2i)(3 - 2i)$
42. $(1 + i)^3$

43. $(1 + 2i)^2(1 - 2i)^2$

44. i^{29}

45. $\sqrt{-16}$

46. $\dfrac{2 + 3i}{1 - 5i}$

In Exercises 47 and 48, solve the equation.

47. $x^2 - 6x + 13 = 0$

48. $x^2 - 2x + 4 = 0$

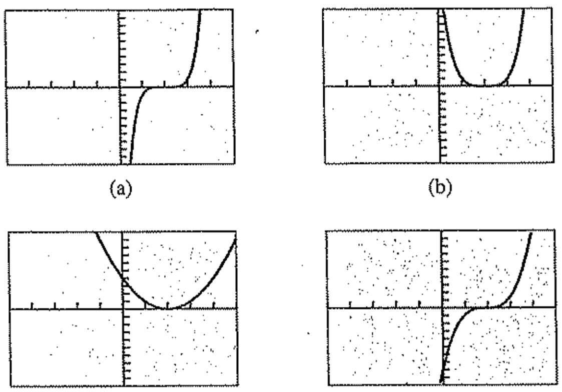
In Exercises 49–52, match the polynomial function with its graph. Explain your choice.

49. $f(x) = (x - 2)^2$

50. $f(x) = (x - 2)^3$

51. $f(x) = (x - 2)^4$

52. $f(x) = (x - 2)^5$



(a) (b) (c) (d)

In Exercises 53–56, find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational. State the number of nonreal complex zeros.

53. $f(x) = x^4 - 10x^3 + 23x^2$

54. $k(t) = t^4 - 7t^2 + 12$

55. $h(x) = x^3 - 2x^2 - 8x + 5$

56. $k(x) = x^4 - x^3 - 14x^2 + 24x + 5$

In Exercises 57–60, find all of the zeros and write a linear factorization of the function.

57. $f(x) = 2x^3 - 9x^2 + 2x + 30$

58. $f(x) = 5x^3 - 24x^2 + x + 12$

59. $f(x) = 6x^4 + 11x^3 - 16x^2 - 11x + 10$

60. $f(x) = x^4 - 8x^3 + 27x^2 - 50x + 50$, given that $1 + 2i$ is a zero.

In Exercises 61–64, write the function as a product of linear and irreducible quadratic factors all with real coefficients.

61. $f(x) = x^3 - x^2 - x - 2$

62. $f(x) = 9x^3 - 3x^2 - 13x - 1$

63. $f(x) = 2x^4 - 9x^3 + 23x^2 - 31x + 15$

64. $f(x) = 3x^4 - 7x^3 - 3x^2 + 17x + 10$

In Exercises 65–70, write a polynomial function with real coefficients whose zeros and their multiplicities include those listed.

65. Degree 3; zeros: $\sqrt{5}, -\sqrt{5}, 3$

66. Degree 2; -3 only real zero

67. Degree 4; zeros: $3, -2, 1/3, -1/2$

68. Degree 3; zeros: $1 + i, 2$

69. Degree 4; zeros: -2(multiplicity 2), 4(multiplicity 2)

70. Degree 3; zeros: $2 - i, -1$, and $f(2) = 6$

In Exercises 71 and 72, describe how the graph of the given function can be obtained by transforming the graph of the reciprocal function $f(x) = 1/x$. Identify the horizontal and vertical asymptotes.

71. $f(x) = \dfrac{-x + 7}{x - 5}$

72. $f(x) = \dfrac{3x + 5}{x + 2}$

In Exercises 73–76, find the asymptotes and intercepts of the function, and graph it.

73. $f(x) = \dfrac{x^2 + x + 1}{x^2 - 1}$

74. $f(x) = \dfrac{2x^2 + 7}{x^2 + x - 6}$

75. $f(x) = \dfrac{x^2 - 4x + 5}{x + 3}$

76. $g(x) = \dfrac{x^2 - 3x - 7}{x + 3}$

In Exercises 77–84, solve the equation or inequality algebraically, and support graphically.

77. $2x + \dfrac{12}{x} = 11$

78. $\dfrac{x}{x + 2} + \dfrac{5}{x - 3} = \dfrac{25}{x^2 - x - 6}$

79. $2x^3 + 3x^2 - 17x - 30 < 0$

80. $3x^4 + x^3 - 36x^2 + 36x + 16 \geq 0$

81. $\dfrac{x + 3}{x^2 - 4} \geq 0$

82. $\dfrac{x^2 - 7}{x^2 - x - 6} < 1$

83. $(2x - 1)^2|x + 3| \leq 0$

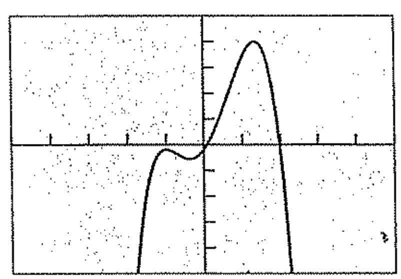
84. $\dfrac{(x - 1)|x - 4|}{\sqrt{x + 3}} > 0$

85. Plot $-3 - 2i$ in the complex plane.

86. Writing to Learn Determine whether

$$f(x) = x^5 - 10x^4 - 3x^3 + 28x^2 + 20x - 2$$

has a zero outside the viewing window. Explain.



[–5, 5] by [–50, 50]

87. **Launching a Rock** Larry uses a slingshot to launch a rock straight up from a point 6 ft above level ground with an initial velocity of 170 ft/sec.

(a) Find an equation that models the height of the rock t seconds after it is launched and graph the equation. (See Example 7 in Section 2.1.)

(b) What is the maximum height of the rock? When will it reach that height?

(c) When will the rock hit the ground?

88. **Volume of a Box** Edgardo Paper Co. has contracted to manufacture a box with no top that is to be made by removing squares of width x from the corners of a 30-in. by 70-in. piece of cardboard.

(a) Find an equation that models the volume of the box.

(b) Determine x so that the box has a volume of 5800 in.^{3}.

89. **Architectural Engineering** Donoma, an engineer at J. P. Cook, Inc., completes structural specifications for a 255-ft-long steel beam anchored between two pilings 50 ft above ground, as shown in the figure. She knows that when a 250-lb object is placed d feet from the west piling, the beam bends s feet where

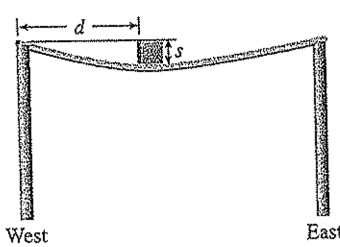
$$s = (8.5 \times 10^{-7})d^2(255 - d).$$

(a) Graph the function s.

(b) What are the dimensions of a viewing window that shows a graph for the values that make sense in this problem situation?

(c) What is the greatest amount of vertical deflection s, and where does it occur?

(d) **Writing to Learn** Give a possible scenario explaining why the solution to (c) does not occur at the halfway point.

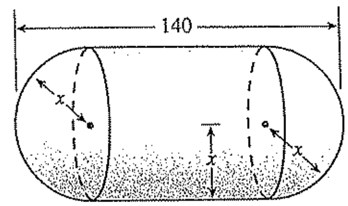


90. **Storage Container** A liquid storage container on a truck is in the shape of a cylinder with hemispheres on each end as shown in the figure. The cylinder and hemispheres have the same radius. The total length of the container is 140 ft.

(a) Determine the volume V of the container as a function of the radius x.

(b) Graph the function $y = V(x)$.

(c) What is the radius of the container with the largest possible volume? What is the volume?



91. **Dow Strategy** A *Dog* stock is one of the 10 Dow stocks with highest dividend yield at the end of a year compared with their stock price. Similarly, a *Star* stock is one of the 10 Dow stocks with lowest dividend yield at the end of a year compared with their stock price. Table 2.25 shows the annual performance for the following year for Dog and Star stocks for several years. Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth.

Table 2.25 Dog and Star Dow Stock Performace

Year	Dog Performance (%)	Star Performance (%)
1996	19.8	27.9
1997	20.4	6.2
1998	9.5	17.5
1999	−2.4	22.4
2000	−1.6	1.7

Source: www.dogsofthedow.com as reported in the USA Today, Februrary 21, 2000.

(a) Find a cubic regression model for the Dog stocks, and graph it together with a scatter plot of the Dog data.

(b) Find a cubic regression model for the Star stocks, and graph it together with a scatter plot of the Star data.

(c) **Writing to Learn** Determine the end behavior (lim) of the two regression models. What do the end behavior models say about using the Dog or Star investment strategy.

92. **National Institute of Health Spending** Table 2.26 shows the spending at the National Institute of Health for several years. Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth.

Table 2.26 Spending at the National Institute of Health

Year	Amount (billion)
1993	10.32
1994	10.95
1995	11.30
1996	11.93
1997	12.74
1998	13.64
1999	15.65
2000	17.91

Source: National Institute of Health as reported in The Chronicle of Higher Education November 26, 1999.

(a) Find a linear regression model, and graph it together with a scatter plot of the data.

(b) Find a quadratic regression model, and graph it together with a scatter plot of the data.

(c) Use the linear and quadratic regression models to estimate when the amount of spending will exceed \$20 billion.

93. **Breaking Even** Midtown Sporting Goods has determined that it needs to sell its soccer shinguards for \$5.25 a pair in order to be competitive. It costs \$4.32 to produce each pair of shinguards, and the weekly overhead cost is \$4000.

(a) Express the average cost that includes the overhead of producing one shinguard as a function of the number x of shinguards produced each week.

(b) Solve algebraically to find the number of shinguards that must be sold each week to make \$8000 in profit. Support your work graphically.

94. **Deer Population** The number of deer P at any time t (in years) in a federal game reserve is given by

$$P(t) = \frac{800 + 640t}{20 + 0.8t}.$$

(a) Find the number of deer when t is 15, 70, and 100.

(b) Find the horizontal asymptote of the graph of $y = P(t)$.

(c) According to the model, what is the largest possible deer population?

95. **Resistors** The total electrical resistance R of two resistors connected in parallel with resistances R_1 and R_2 is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

The total resistance is 1.2 ohms. Let $x = R_1$.

(a) Express the second resistance R_2 as a function of x.

(b) Find R_2 if R_1 is 3 ohms.

96. **Acid Mixture** Suppose that x ounces of distilled water are added to 50 oz of pure acid.

(a) Express the concentration $C(x)$ of the new mixture as a function of x.

(b) Use a graph to determine how much distilled water should be added to the pure acid to produce a new solution that is less than 60% acid.

(c) Solve (b) algebraically.

97. **Industrial Design** Johnson Cannery will pack peaches in 1-L cylindrical cans. Let x be the radius of the base of the can in centimeters.

(a) Express the surface area S of the can as a function of x.

(b) Find the radius and height of the can if the surface area is 900 cm^2.

(c) What dimensions are possible for the can if the surface area is to be less than 900 cm^2?

98. **Industrial Design** Gilman Construction is hired to build a rectangular tank with a square base and no top. The tank is to hold 1000 ft^3 of water. Let x be a length of the base.

(a) Express the outside surface area S of the tank as a function of x.

(b) Find the length, width, and height of the tank if the outside surface area is 600 ft^2.

(c) What dimensions are possible for the tank if the outside surface area is to be less than 600 ft^2?