Quick Review 5.5

In Exercises 1–4, solve the equation $a/b = c/d$ for the given variable.

1. a **2.** b

3. c **4.** d

In Exercises 5 and 6, evaluate the expression.

5. $\dfrac{7 \sin 48°}{\sin 23°}$ **6.** $\dfrac{9 \sin 121°}{\sin 14°}$

In Exercises 7–10, solve for the angle x.

7. $\sin x = 0.3, \quad 0° < x < 90°$

8. $\sin x = 0.3, \quad 90° < x < 180°$

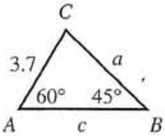
9. $\sin x = -0.7, \quad 180° < x < 270°$

10. $\sin x = -0.7, \quad 270° < x < 360°$

Section 5.5 Exercises

In Exercises 1–4, solve the triangle.

1.



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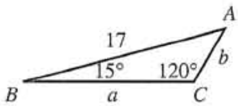

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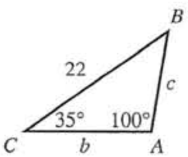

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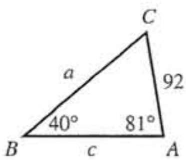


In Exercises 5–8, solve the triangle.

5. $A = 40°, \quad B = 30°, \quad b = 10$

6. $A = 50°, \quad B = 62°, \quad a = 4$

7. $A = 33°, \quad B = 70°, \quad b = 7$

8. $B = 16°, \quad C = 103°, \quad c = 12$

In Exercises 9–12, solve the triangle.

9. $A = 32°, \quad a = 17, \quad b = 11$

10. $A = 49°, \quad a = 32, \quad b = 28$

11. $B = 70°, \quad b = 14, \quad c = 9$

12. $C = 103°, \quad b = 46, \quad c = 61$

In Exercises 13–18, state whether the given measurements determine zero, one, or two triangles.

13. $A = 36°, \quad a = 2, \quad b = 7$

14. $B = 82°, \quad b = 17, \quad c = 15$

15. $C = 36°, \quad a = 17, \quad c = 16$

16. $A = 73°, \quad a = 24, \quad b = 28$

17. $C = 30°, \quad a = 18, \quad c = 9$

18. $B = 88°, \quad b = 14, \quad c = 62$

In Exercises 19–22, two triangles can be formed using the given measurements. Solve both triangles.

19. $A = 64°, \quad a = 16, \quad b = 17$

20. $B = 38°, \quad b = 21, \quad c = 25$

21. $C = 68°, \quad a = 19, \quad c = 18$

22. $B = 57°, \quad a = 11, \quad b = 10$

23. Determine the values of b that will produce the given number of triangles if $a = 10$ and $B = 42°$.

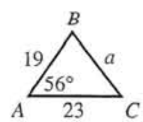
(a) two triangles **(b)** one triangle **(c)** zero triangles

24. Determine the values of c that will produce the given number of triangles if $b = 12$ and $C = 53°$.

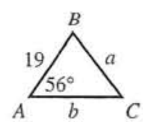
(a) two triangles **(b)** one triangle **(c)** zero triangles

In Exercises 25 and 26, decide whether the triangle can be solved using the Law of Sines. If so, solve it. If not, explain why not.

25.

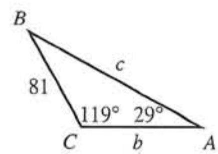


(a)

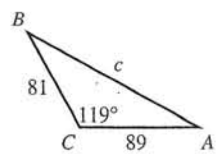


(b)

26.



(a)



(b)

In Exercises 27–36, respond in one of the following ways:

(a) State, "Cannot be solved with the Law of Sines."

(b) State, "No triangle is formed."

(c) Solve the triangle.

27. $A = 61°, \quad a = 8, \quad b = 21$

28. $B = 47°, \quad a = 8, \quad b = 21$

29. $A = 136°, \quad a = 15, \quad b = 28$

30. $C = 115°, \quad b = 12, \quad c = 7$

31. $B = 42°, \quad c = 18, \quad C = 39°$

32. $A = 19°$, $b = 22$, $B = 47°$

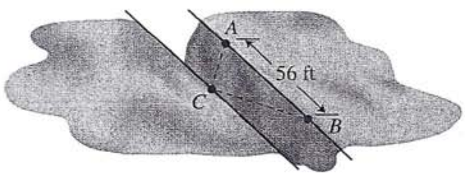
33. $C = 75°$, $b = 49$, $c = 48$

34. $A = 54°$, $a = 13$, $b = 15$

35. $B = 31°$, $a = 8$, $c = 11$

36. $C = 65°$, $a = 19$, $b = 22$

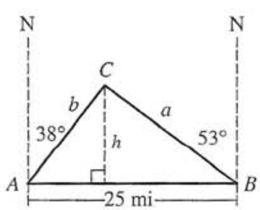
37. Surveying a Canyon Two markers A and B on the same side of a canyon rim are 56 ft apart. A third marker C, located across the rim, is positioned so that $\angle BAC = 72°$ and $\angle ABC = 53°$.



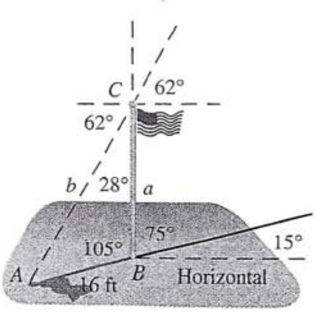
(a) Find the distance between C and A.

(b) Find the distance between the two canyon rims. (Assume they are parallel.)

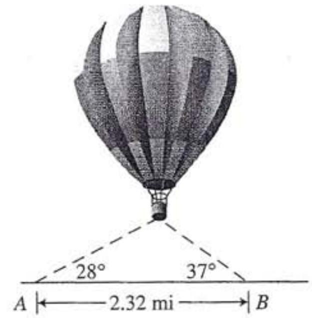
38. Weather Forecasting Two meteorologists are 25 mi apart located on an east-west road. The meteorologist at point A sights a tornado 38° east of north. The meteorologist at point B sights the same tornado at 53° west of north. Find the distance from each meteorologist to the tornado. Also find the distance between the tornado and the road.



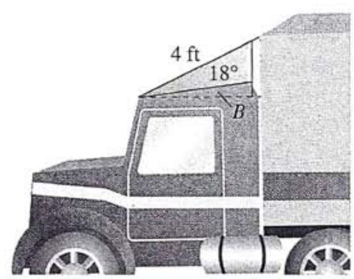
39. Engineering Design A vertical flagpole stands beside a road that slopes at an angle of 15° with the horizontal. When the angle of elevation of the sun is 62°, the flagpole casts a 16-ft shadow downhill along the road. Find the height of the flagpole.



40. Altitude Observers 2.32 mi apart see a hot-air balloon directly between them but at the angles of elevation shown in the figure. Find the altitude of the balloon.



41. Reducing Air Resistance A 4-ft airfoil attached to the cab of a truck reduces wind resistance. If the angle between the airfoil and the cab top is 18° and angle B is 10°, find the length of a vertical brace positioned as shown in the figure.

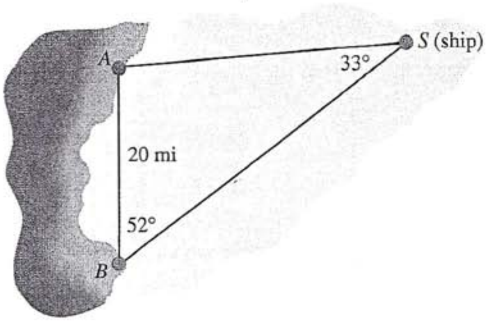


42. Group Activity Ferris Wheel Design A Ferris wheel has 16 evenly spaced cars. The distance between adjacent chairs is 15.5 ft. Find the radius of the wheel (to the nearest 0.1 ft.)

43. Finding Height Two observers are 600 ft apart on opposite sides of a flagpole. The angles of elevation from the observers to the top of the pole are 19° and 21°. Find the height of the flagpole.

44. Finding Height Two observers are 400 ft apart on opposite sides of a tree. The angles of elevation from the observers to the top of the tree are 15° and 20°. Find the height of the tree.

45. Finding Distance Two lighthouses A and B are known to be exactly 20 mi apart on a north-south line. A ship's captain at S measures $\angle ASB$ to be 33°. A radio operator at B measures $\angle ABS$ to be 52°. Find the distance from the ship to each lighthouse.



Explorations

46. Writing to Learn

(a) Show that there are infinitely many triangles with AAA given if the sum of the three positive angles is 180°.

(b) Give three examples of triangles where $A = 30°$, $B = 60°$, and $C = 90°$.

(c) Give three examples where $A = B = C = 60°$.

47. Use the Law of Sines and the cofunction identities to derive the following formulas from right triangle trigonometry:

(a) $\sin A = \frac{opp}{hyp}$ **(b)** $\cos A = \frac{adj}{hyp}$ **(c)** $\tan A = \frac{opp}{adj}$

48. Wrapping up Exploration 1 Refer to Figures 5.17 and 5.18 in Exploration 1 of this section.

(a) Express h in terms of angle A and length AB.

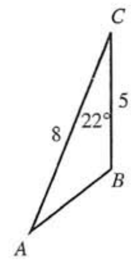
(b) In terms of the given angle A and the given length AB, state the conditions on length BC that will result in no triangle being formed.

(c) In terms of the given angle A and the given length AB, state the conditions on length BC that will result in a unique triangle being formed.

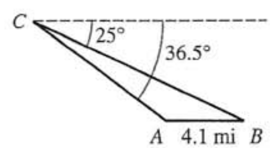
(d) In terms of the given angle A and the given length AB, state the conditions on length BC that will result in two possible triangles being formed.

Extending the Ideas

49. Solve this triangle assuming that $\angle B$ is obtuse. (*Hint*: Draw a perpendicular from A to the line through B and C.)



50. Pilot Calculations Towers A and B are known to be 4.1 mi apart on level ground. A pilot measures the angles of depression to the towers to be 36.5° and 25°, respectively, as shown in the figure. Find distances AC and BC and the height of the airplane.



5.6 The Law of Cosines

Deriving the Law of Cosines • Solving Triangles (SAS, SSS) • Triangle Area and Heron's Formula • Applications

Deriving the Law of Cosines

Having seen the Law of Sines, you will probably not be surprised to learn that there is a Law of Cosines. There are many such parallels in mathematics. What you might find surprising is that the Law of Cosines has absolutely no resemblance to the Law of Sines. Instead, it resembles the Pythagorean Theorem. In fact, the Law of Cosines is often called the "generalized Pythagorean Theorem" because it contains that classic theorem as a special case.

Quick Review 5.6

In Exercises 1–4, find an angle between 0° and 180° that is a solution to the equation.

1. $\cos A = 3/5$

2. $\cos C = -0.23$

3. $\cos A = -0.68$

4. $3 \cos C = 1.92$

In Exercises 5 and 6, solve the equation (in terms of x and y) for **(a)** $\cos A$ and **(b)** A, $0 \le A \le 180°$.

5. $9^2 = x^2 + y^2 - 2xy \cos A$

6. $y^2 = x^2 + 25 - 10 \cos A$

In Exercises 7–10, find a quadratic polynomial with real coefficients that satisfies the given condition.

7. Has two positive zeros

8. Has one positive and one negative zero

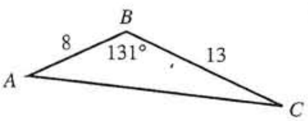
9. Has no real zeros

10. Has exactly one positive zero

Section 5.6 Exercises

In Exercises 1–4, solve the triangle.

1.



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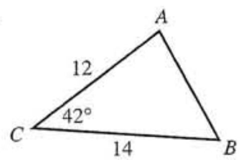

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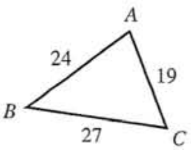
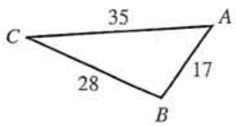


4.



In Exercises 5–16, solve the triangle.

5. $A = 55°$, $b = 12$, $c = 7$

6. $B = 35°$, $a = 43$, $c = 19$

7. $a = 12$, $b = 21$, $C = 95°$

8. $b = 22$, $c = 31$, $A = 82°$

9. $a = 1$, $b = 5$, $c = 4$

10. $a = 1$, $b = 5$, $c = 8$

11. $a = 3.2$, $b = 7.6$, $c = 6.4$

12. $a = 9.8$, $b = 12$, $c = 23$

13. $A = 42°$, $a = 7$, $b = 10$

14. $A = 57°$, $a = 11$, $b = 10$

15. $A = 63°$, $a = 8.6$, $b = 11.1$

16. $A = 71°$, $a = 9.3$, $b = 8.5$

In Exercises 17–20, find the area of the triangle.

17. $A = 47°$, $b = 32$ ft, $c = 19$ ft

18. $A = 52°$, $b = 14$ m, $c = 21$ m

19. $B = 101°$, $a = 10$ cm, $c = 22$ cm

20. $C = 112°$, $a = 1.8$ in., $b = 5.1$ in.

In Exercises 21–28, decide whether a triangle can be formed with the given side lengths. If so, use Heron's formula to find the area of the triangle.

21. $a = 4$, $b = 5$, $c = 8$

22. $a = 5$, $b = 9$, $c = 7$

23. $a = 3$, $b = 5$, $c = 8$

24. $a = 23$, $b = 19$, $c = 12$

25. $a = 19.3$, $b = 22.5$, $c = 31$

26. $a = 8.2$, $b = 12.5$, $c = 28$

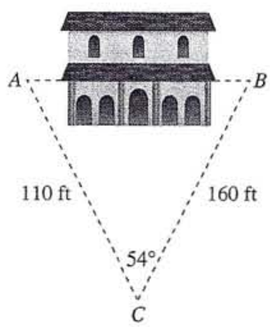
27. $a = 33.4$, $b = 28.5$, $c = 22.3$

28. $a = 18.2$, $b = 17.1$, $c = 12.3$

29. Find the radian measure of the largest angle in the triangle with sides of 4, 5, and 6.

30. A parallelogram has sides of 18 and 26 ft, and an angle of 39°. Find the shorter diagonal.

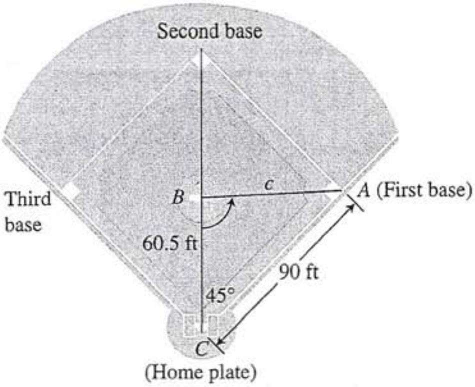
31. Measuring Distance Indirectly Juan wants to find the distance between two points A and B on opposite sides of a building. He locates a point C that is 110 ft from A and 160 ft from B, as illustrated in the figure. If the angle at C is 54°, find distance AB.



32. Designing a Baseball Field

(a) Find the distance from the center of the front edge of the pitcher's rubber to the far corner of second base. How does this distance compare with the distance from the pitcher's rubber to first base? (See Example 6.)

(b) Find $\angle B$ in $\triangle ABC$.

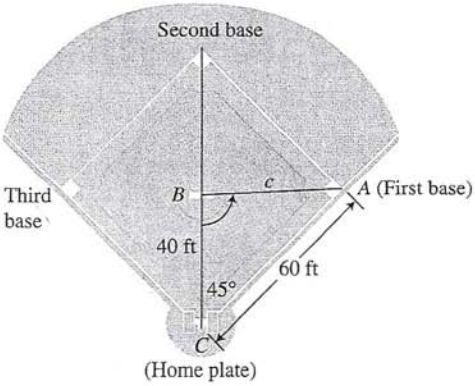


33. Designing a Softball Field In softball, adjacent bases are 60 ft apart. The distance from the center of the front edge of the pitcher's rubber to the far corner of home plate is 40 ft.

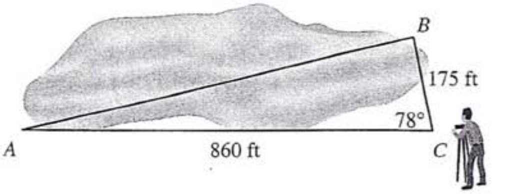
(a) Find the distance from the center of the pitcher's rubber to the far corner of first base.

(b) Find the distance from the center of the pitcher's rubber to the far corner of second base.

(c) Find $\angle B$ in $\triangle ABC$.



34. Surveyor's Calculations Tony must find the distance from A to B on opposite sides of a lake. He locates a point C that is 860 ft from A and 175 ft from B. He measures the angle at C to be 78°. Find distance AB.

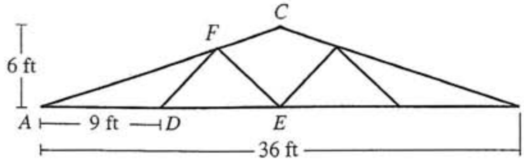


35. Construction Engineering A manufacturer is designing the roof truss that is modeled in the figure shown.

(a) Find the measure of $\angle CAE$.

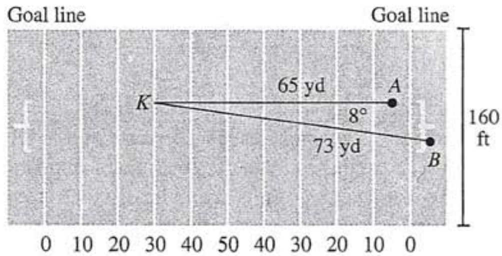
(b) If $AF = 12$ ft, find the length DF.

(c) Find the length EF.

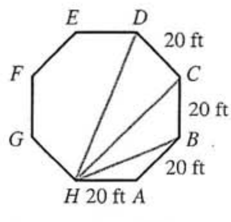


36. Navigation Two airplanes flying together in formation take off in different directions. One flies due east at 350 mph, and the other flies east-northeast at 380 mph. How far apart are the two airplanes 2 h after they separate, assuming that they fly at the same altitude?

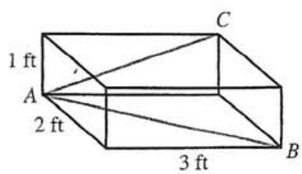
37. Football Kick The player waiting to receive a kickoff stands at the 5 yard line (point A) as the ball is being kicked 65 yd up the field from the opponent's 30 yard line. The kicked ball travels 73 yd at an angle of 8° to the right of the receiver, as shown in the figure (point B). Find the distance the receiver runs to catch the ball.



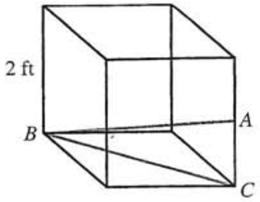
38. Architectural Design Building Inspector Julie Wang checks a building in the shape of a regular octagon, each side 20 ft long. She checks that the contractor has located the corners of the foundation correctly by measuring several of the diagonals. Calculate what the lengths of diagonals HB, HC, and HD should be.



39. Group Activity Connecting Trigonometry and Geometry $\angle CAB$ is inscribed in a rectangular box whose sides are 1, 2 and 3 ft long as shown. Find the measure of $\angle CAB$.



40. Group Activity Connecting Trigonometry and Geometry A cube has edges of length 2 ft. Point A is the midpoint of an edge. Find the measure of $\angle ABC$.



Explorations

41. Find the area of a regular polygon with n sides inscribed inside a circle of radius r. (Express your answer in terms of n and r.)

42. **(a)** Prove the identity: $\dfrac{\cos A}{a} = \dfrac{b^2 + c^2 - a^2}{2abc}$.

(b) Prove the (tougher) identity:

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}.$$

[*Hint*: use the identity in (a), along with its other variations.]

43. Navigation Two ships leave a common port at 8:00 A.M. and travel at a constant rate of speed. Each ship keeps a log showing its distance from port and its distance from the other ship. Portions of the logs from later that morning for both ships are shown below.

Time	Naut mi from port	Naut mi from ship B
9:00	15.1	8.7
10:00	30.2	17.3

Time	Naut mi from port	Naut mi from ship A
9:00	12.4	8.7
11:00	37.2	26.0

(a) How fast is each ship traveling? (Express your answer in knots, which are nautical miles per hour.)

(b) What is the angle of intersection of the courses of the two ships?

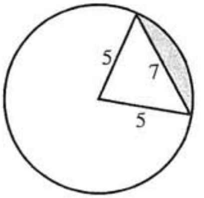
(c) How far apart are the ships at 12:00 noon if they maintain the same courses and speeds? ■

Extending the Ideas

44. Prove that the area of a triangle can be found with the formula

$$\triangle \text{ Area} = \frac{a^2 \sin B \sin C}{2 \sin A}.$$

45. A **segment** of a circle is the region enclosed between a chord of a circle and the arc intercepted by the chord. Find the area of a segment intercepted by a 7-inch chord in a circle of radius 5 inches.



Law of Cosines

In any $\triangle ABC$ with angles A, B, and C opposite sides a, b, and c respectively, the following equations are true:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Triangle Area

The area of any $\triangle ABC$ with angles A, B, and C opposite sides a, b, and c respectively, is given by any of the following formulas:

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C.$$

Heron's Formula

Let a, b, and c be the sides of $\triangle ABC$, and let s denote the semiperimeter $\dfrac{a+b+c}{2}$.

Then the area of $\triangle ABC$ is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$.

Procedures

Strategies for Proving an Identity

1. Begin with the more complicated expression and work toward the less complicated expression.
2. If no other move suggests itself, convert the entire expression to one involving sines and cosines.
3. Combine fractions by writing them over a common denominator.
4. Use the algebraic identity $(a + b)(a - b) = a^2 - b^2$ to set up applications of the Pythagorean identities.
5. Always be mindful of the "target" expression, and favor manipulations that bring you closer to your goal.

Chapter 5 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1 and 2, write the expression as the sine, cosine, or tangent of an angle.

1. $2 \sin 100° \cos 100°$

2. $\dfrac{2 \tan 40°}{1 - \tan^2 40°}$

In Exercises 3 and 4, simplify the expression to a single term. Support your answer graphically.

3. $(1 - 2 \sin^2 \theta)^2 + 4 \sin^2 \theta \cos^2 \theta$

4. $1 - 4 \sin^2 x \cos^2 x$

In Exercises 5–22, prove the identity.

5. $\cos 3x = 4 \cos^3 x - 3 \cos x$

6. $\cos^2 2x - \cos^2 x = \sin^2 x - \sin^2 2x$

7. $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

8. $2 \sin \theta \cos^3 \theta + 2 \sin^3 \theta \cos \theta = \sin 2\theta$

9. $\csc x - \cos x \cot x = \sin x$

10. $\dfrac{\tan \theta + \sin \theta}{2 \tan \theta} = \cos^2\left(\dfrac{\theta}{2}\right)$

11. $\dfrac{1 + \tan \theta}{1 - \tan \theta} + \dfrac{1 + \cot \theta}{1 - \cot \theta} = 0$

12. $\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$

13. $\cos^2\left(\dfrac{t}{2}\right) = \dfrac{1 + \sec t}{2 \sec t}$

14. $\dfrac{\tan^3 \gamma - \cot^3 \gamma}{\tan^2 \gamma + \csc^2 \gamma} = \tan \gamma - \cot \gamma$

15. $\dfrac{\cos \phi}{1 - \tan \phi} + \dfrac{\sin \phi}{1 - \cot \phi} = \cos \phi + \sin \phi$

16. $\dfrac{\cos(-z)}{\sec(-z) + \tan(-z)} = 1 + \sin z$

17. $\sqrt{\dfrac{1-\cos y}{1+\cos y}} = \dfrac{1-\cos y}{|\sin y|}$

18. $\sqrt{\dfrac{1-\sin \gamma}{1+\sin \gamma}} = \dfrac{|\cos \gamma|}{1+\sin \gamma}$

19. $\tan\left(u+\dfrac{3\pi}{4}\right) = \dfrac{\tan u - 1}{1+\tan u}$

20. $\dfrac{1}{4}\sin 4\gamma = \sin\gamma\cos^3\gamma - \cos\gamma\sin^3\gamma$

21. $\tan\dfrac{1}{2}\beta = \csc\beta - \cot\beta$

22. $\arctan t = \dfrac{1}{2}\arctan\dfrac{2t}{1-t^2}, \quad -1 < t < 1$

In Exercises 23 and 24, use a grapher to conjecture whether the equation is likely to be an identity. Confirm your conjecture.

23. $\sec x - \sin x \tan x = \cos x$

24. $(\sin^2\alpha - \cos^2\alpha)(\tan^2\alpha + 1) = \tan^2\alpha - 1$

In Exercises 25–28, write the expression in terms of $\sin x$ and $\cos x$ only.

25. $\sin 3x + \cos 3x$

26. $\sin 2x + \cos 3x$

27. $\cos^2 2x - \sin 2x$

28. $\sin 3x - 3\sin 2x$

In Exercises 29–34, find the general solution without using a calculator. Give exact answers.

29. $\sin 2x = 0.5$

30. $\cos x = \dfrac{\sqrt{3}}{2}$

31. $\tan x = -1$

32. $2\sin^{-1} x = \sqrt{2}$

33. $\tan^{-1} x = 1$

34. $2\cos 2x = 1$

In Exercises 35–38, solve the equation graphically.

35. $\sin^2 x - 3\cos x = -0.5$

36. $\cos^3 x - 2\sin x - 0.7 = 0$

37. $\sin^4 x + x^2 = 2$

38. $\sin 2x = x^3 - 5x^2 + 5x + 1$

In Exercises 39–44, find all solutions in the interval $[0, 2\pi)$ without using a calculator. Give exact answers.

39. $2\cos x = 1$

40. $\sin 3x = \sin x$

41. $\sin^2 x - 2\sin x - 3 = 0$

42. $\cos 2t = \cos t$

43. $\sin(\cos x) = 1$

44. $\cos 2x + 5\cos x = 2$

In Exercises 45–48, solve the inequality. Use any method, but give exact answers.

45. $2\cos 2x > 1$ for $0 \le x < 2\pi$

46. $\sin 2x > 2\cos x$ for $0 < x \le 2\pi$

47. $2\cos x < 1$ for $0 \le x < 2\pi$

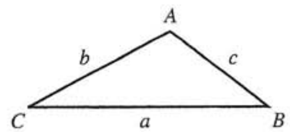
48. $\tan x < \sin x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

In Exercises 49 and 50, find an equivalent equation of the form $y = a\sin(bx + c)$. Support your work graphically.

49. $y = 3\sin 3x + 4\cos 3x$

50. $y = 5\sin 2x - 12\cos 2x$

In Exercises 51–58, solve $\triangle ABC$.



51. $A = 79°, \quad B = 33°, \quad a = 7$

52. $a = 5, \quad b = 8, \quad B = 110°$

53. $a = 8, \quad b = 3, \quad B = 30°$

54. $a = 14.7, \quad A = 29.3°, \quad C = 33°$

55. $A = 34°, \quad B = 74°, \quad c = 5$

56. $c = 41, \quad A = 22.9°, \quad C = 55.1°$

57. $a = 5, \quad b = 7, \quad c = 6$

58. $A = 85°, \quad a = 6, \quad b = 4$

In Exercises 59 and 60, find the area of $\triangle ABC$.

59. $a = 3, \quad b = 5, \quad c = 6$

60. $a = 10, \quad b = 6, \quad C = 50°$

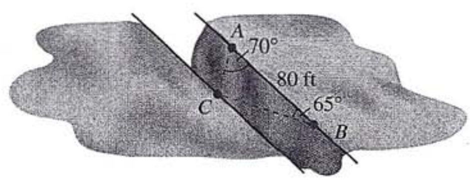
61. If $a = 12$ and $B = 28°$, determine the values of b that will produce the indicated number of triangles

(a) Two (b) One (c) Zero

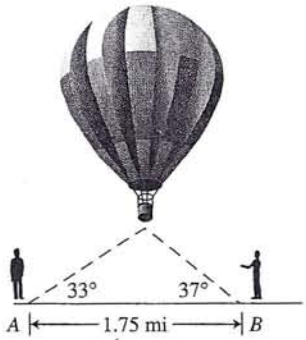
62. **Surveying a Canyon** Two markers A and B on the same side of a canyon rim are 80 ft apart, as shown in the figure. A hiker is located across the rim at point C. A surveyor determines that $\angle BAC = 70°$ and $\angle ABC = 65°$.

(a) What is the distance between the hiker and point A?

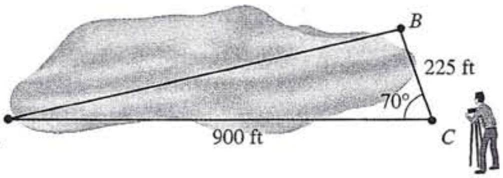
(b) What is the distance between the two canyon rims? (Assume they are parallel.)



63. **Altitude** A hot-air balloon is seen over Tucson, Arizona, simultaneously by two observers at points A and B that are 1.75 mi apart on level ground and in line with the balloon. The angles of elevation are as shown here. How high above ground is the balloon?



64. Finding Distance In order to determine the distance between two points A and B on apposite sides of a lake, a surveyor chooses a point C that is 900 ft from A and 225 ft from B, as shown in the figure. If the measure of the angle at C is 70°, find the distance between A and B.



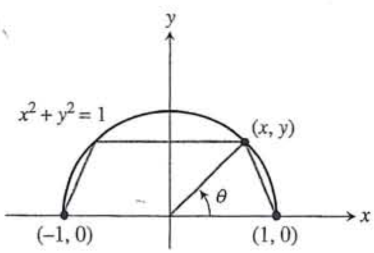
65. Finding Radian Measure Find the radian measure of the largest angle of the triangle whose sides have lengths 8, 9, and 10.

66. Finding a Parallelogram A parallelogram has sides of 15 and 24 ft, and an angle of 40°. Find the diagonals.

67. Maximizing Area A trapezoid is inscribed in the upper half of a unit circle, as shown in the figure.

(a) Write the area of the trapezoid as a function of θ.

(b) Find the value of θ that maximizes the area of the trapezoid and the maximum area.



68. Beehive Cells A single cell in a beehive is a regular hexagonal prism open at the front with a trihedral cut at the back. Trihedral refers to a vertex formed by three faces of a polyhedron. It can be shown that the surface area of a cell is given by

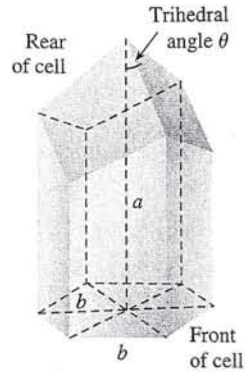
$$S(\theta) = 6ab + \frac{3}{2}b^2\left(-\cot\theta + \frac{\sqrt{3}}{\sin\theta}\right),$$

where θ is the angle between the axis of the prism and one of the back faces, a is the depth of the prism, and b is the length of the hexagonal front. Assume $a = 1.75$ in. and $b = 0.65$ in.

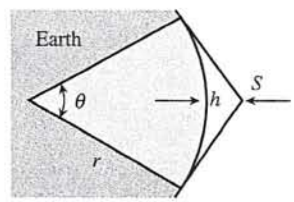
(a) Graph the function $y = S(\theta)$.

(b) What value of θ gives the minimum surface area? (*Note:* This answer is quite close to the observed angle in nature.)

(c) What is the minimum surface area?



69. Cable Television Coverage A cable broadcast satellite S orbits a planet at a height h (in miles) above the earth's surface, as shown in the figure. The two lines from S are tangent to the earth's surface. The part of the earth's surface that is in the broadcast area of the satellite is determined by the central angle θ indicated in the figure.



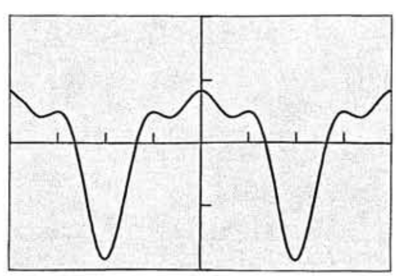
(a) Assuming that the earth is spherical with a radius of 4000 mi, write h as a function of θ.

(b) Approximate θ for a satellite 200 mi above the surface of the earth.

70. Finding Extremum Values The graph of

$$y = \cos x - \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x$$

is shown in the figure. The x-values that correspond to local maximum and minimum points are solutions of the equation $\sin x - \sin 2x + \sin 3x = 0$. Solve this equation algebraically, and support your solution using the graph of y.



$[-2\pi, 2\pi]$ by $[-2, 2]$

71. Using Trigonometry in Geometry A regular hexagon whose sides are 16 cm is inscribed in a circle. Find the area inside the circle and outside the hexagon.

72. Using Trigonometry in Geometry A circle is inscribed in a regular pentagon whose sides are 12 cm. Find the area inside the pentagon and outside the circle.

73. Using Trigonometry in Geometry A wheel of cheese in the shape of a right circular cylinder is 18 cm in diameter and 5 cm thick. If a wedge of cheese with a central angle of 15° is cut from the wheel, find the volume of the cheese wedge.

74. Product-to-Sum Formulas Prove the following identities, which are called the **product-to-sum formulas**.

(a) $\sin u \sin v = \frac{1}{2}(\cos(u - v) - \cos(u + v))$

(b) $\cos u \cos v = \frac{1}{2}(\cos(u - v) + \cos(u + v))$

(c) $\sin u \cos v = \frac{1}{2}(\sin(u + v) + \sin(u - v))$

75. Sum-to-Product Formulas Use the product-to-sum formulas in Exercise 74 to prove the following identities, which are called the **sum-to-product formulas.**

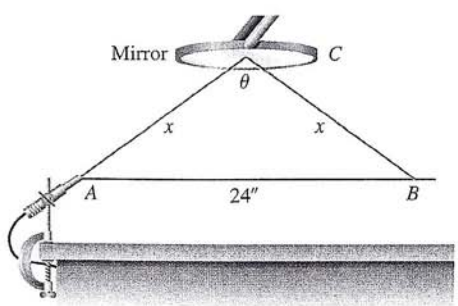
(a) $\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2}$

(b) $\sin u - \sin v = 2 \sin \frac{u-v}{2} \cos \frac{u+v}{2}$

(c) $\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2}$

(d) $\cos u - \cos v = -2 \sin \frac{u+v}{2} \sin \frac{u-v}{2}$

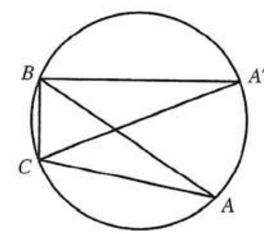
76. Catching Students Faking Data Carmen and Pat both need to make up a missed physics lab. They are to measure the total distance ($2x$) traveled by a beam of light from point A to point B and record it in 20° increments of θ as they adjust the mirror (C) upward vertically.



They report the following measurements. However, only one of the students actually did the lab; the other skipped it and faked the data. Who faked the data, and how can you tell?

CARMEN		PAT	
θ	$2x$	θ	$2x$
160°	24.4″	160°	24.5″
140°	25.6″	140°	25.2″
120°	28.0″	120°	26.4″
100°	31.2″	100°	30.4″
80°	37.6″	80°	35.2″
60°	48.0″	60°	48.0″
40°	70.4″	40°	84.0″
20°	138.4″	20°	138.4″

77. An Interesting Fact about (sin *A*)/*a* The ratio $(\sin A)/a$ that shows up in the Law of Sines shows up another way in the geometry of $\triangle ABC$: It is the reciprocal of the radius of the circumscribed circle.



(a) Let $\triangle ABC$ be circumscribed as shown in the diagram, and construct diameter CA'. Explain why $\angle A'BC$ is a right angle.

(b) Explain why $\angle A'$ and $\angle A$ are congruent.

(c) If a, b, and c are the sides opposite angles A, B, and C as usual, explain why $\sin A' = a/d$, where d is the diameter of the circle.

(d) Finally, explain why $(\sin A)/a = 1/d$.

(e) Do $(\sin B)/b$ and $(\sin C)/c$ also equal $1/d$? Why?