<u>Geometry - Midyear Exam Study Guide</u>

My Geometry EXAM is on ______________ at ________ in room ________

I MUST bring my own calculator and "cheat" sheet (one side only - you can write whatever you'd like)

Chapter 1 - Tools of Geometry

acute angle (p. 37)
adjacent angles (p. 38)
angle (p. 36)
angle bisector (p. 46)
~~axiom (p. 18)~~)
collinear points (p. 17)
compass (p. 44)
complementary angles (p. 38)
congruent angles (p. 37)
congruent segments (p. 31)
conjecture (p. 5)
construction (p. 44)
coordinate (p. 31)
coplanar (p. 17)
counterexample (p. 5)
~~foundation drawing (p. 11)~~
inductive reasoning (p. 4)
~~isometric drawing (p. 10)~~
line (p. 17)
midpoint (p. 32)
~~net (p. 12)~~
obtuse angle (p. 37)
opposite rays (p. 23)
~~orthographic drawing (p. 11~~)
parallel lines (p. 24)
parallel planes (p. 24)
perpendicular bisector (p. 45)
perpendicular lines (p. 45)
plane (p. 17)
point (p. 17)
postulate (p. 18)
ray (p. 23)
right angle (p. 37)
segment (p. 23)
skew lines (p. 24)
space (p. 17)
straight angle (p. 37)
straightedge (p. 44)
supplementary angles (p. 38)
vertical angles (p. 38)

Postulate 1-1
Through any two points there is exactly one line. (p. 18)

Postulate 1-2
If two lines intersect, then they intersect in exactly one point. (p. 18)

Postulate 1-3
If two planes intersect, then they intersect in exactly one line. (p. 18)

Postulate 1-4
Through any three noncollinear points there is exactly one plane. (p. 19)

Postulate 1-5
Ruler Postulate
The points of a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers. (p. 31)

Postulate 1-6
Segment Addition Postulate
If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$. (p. 32)

Postulate 1-7
Protractor Postulate
Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be opposite rays in a plane. $\overrightarrow{OA}$, $\overrightarrow{OB}$, and all the rays with endpoint O that can be drawn on one side of $\overleftrightarrow{AB}$ can be paired with the real numbers from 0 to 180 so that

a. $\overrightarrow{OA}$ is paired with 0 and $\overrightarrow{OB}$ is paired with 180.

b. If $\overrightarrow{OC}$ is paired with x and $\overrightarrow{OD}$ is paired with y, then $m\angle COD = |x - y|$. (p. 37)

Postulate 1-8
Angle Addition Postulate
If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.
If $\angle AOC$ is a straight angle, then $m\angle AOB + m\angle BOC = 180$. (p. 38)

Postulate 1-9
If two figures are congruent, then their areas are equal. (p. 64)

Postulate 1-10
The area of a region is the sum of the areas of its nonoverlapping parts. (p. 64)

The Midpoint Formula
The coordinates of the midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ are the following.
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (p. 55)

The Distance Formula
The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (p. 53)

- Proof on p. 421, Exercise 34

Chapter 2 - Reasoning and Proof

biconditional (p. 87)
conclusion (p. 80)
conditional (p. 80)
converse (p. 81)
deductive reasoning (p. 94)
hypothesis (p. 80)
Law of Detachment (p. 94)
Law of Syllogism (p. 95)
paragraph proof (p. 111)
Reflexive Property (p. 105)
Symmetric Property (p. 105)
theorem (p. 110)
Transitive Property (p. 105)
truth value (p. 81)

Law of Detachment

If a conditional is true and its hypothesis is true, then its conclusion is true. In symbolic form: If $p \to q$ is a true statement and p is true, then q is true. (p. 95)

Law of Syllogism

If $p \to q$ and $q \to r$ are true statements, then $p \to r$ is a true statement. (p. 95)

Properties of Congruence

Reflexive Property
$\overline{AB} \cong \overline{AB}$ and $\angle A \cong \angle A$

Symmetric Property
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive Property
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. (p. 105)

Theorem 2-1

Vertical Angles Theorem
Vertical angles are congruent. (p. 110)
- Proof on p. 111

Theorem 2-2

Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent. (p. 111)
- Proofs on p. 112, Example 2; p. 114, Exercise 27

Theorem 2-3

Congruent Complements Theorem
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent. (p. 112)
- Proofs on p. 113, Exercise 7; p. 114, Exercise 28

Theorem 2-4

All right angles are congruent. (p. 112)
- Proof on p. 113, Exercise 14

Theorem 2-5

If two angles are congruent and supplementary, then each is a right angle. (p. 112)
- Proof on p. 114, Exercise 21

Chapter 3 - Parallel and Perpendicular Lines

acute triangle (p. 148)
alternate exterior angles (p. 129)
alternate interior angles (p. 127)
concave polygon (p. 158)
convex polygon (p. 158)
corresponding angles (p. 127)
equiangular triangle (p. 148)
equiangular polygon (p. 160)
equilateral triangle (p. 148)
equilateral polygon (p. 160)
exterior angle of a polygon (p. 149)
flow proof (p. 135)
isosceles triangle (p. 148)
obtuse triangle (p. 148)
~~point-slope form (p. 168)~~
polygon (p. 157)
regular polygon (p. 160)
remote interior angles (p. 149)
right triangle (p. 148)
same-side exterior angles (p. 129)
same-side interior angles (p. 127)
scalene triangle (p. 148)
~~slope-intercept form (p. 166)~~
~~standard form of a linear equation (p. 167)~~
transversal (p. 127)
two-column proof (p. 129)

Postulate 3-1
Corresponding Angles Postulate
If a transversal intersects two parallel lines, then corresponding angles are congruent. (p. 128)

Theorem 3-1
Alternate Interior Angles Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent. (p. 128)
- Proof on p. 129

Theorem 3-2
Same-Side Interior Angles Theorem
If a transversal intersects two parallel lines, then same-side interior angles are supplementary. (p. 128)
- Proof on p. 132, Exercise 29

Theorem 3-3
Alternate Exterior Angles Theorem
If a transversal intersects two parallel lines, then alternate exterior angles are congruent. (p. 130)
- Proof on p. 129, Example 3

Theorem 3-4
Same-Side Exterior Angles Theorem
If a transversal intersects two parallel lines, then same-side exterior angles are supplementary. (p. 130)
- Proof on p. 129, Quick Check 3

Postulate 3-2
Converse of the Corresponding Angles Postulate
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel. (p. 134)

Theorem 3-5
Converse of the Alternate Interior Angles Theorem
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel. (p. 135)
- Proof on p. 135

Theorem 3-6
Converse of the Same-Side Interior Angles Theorem
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel. (p. 135)
- Proofs on p. 138, Exercise 22 and p. 139, Exercise 40

Theorem 3-7
Converse of the Alternate Exterior Angles Theorem
If two lines and a transversal form alternate exterior angles that are congruent, then the lines are parallel. (p. 136)
- Proof on p. 136

Theorem 3-8
Converse of the Same-Side Exterior Angles Theorem
If two lines and a transversal form same-side exterior angles that are supplementary, then the lines are parallel. (p. 136)
- Proof on p. 138, Exercise 27

Theorem 3-9
If two lines are parallel to the same line, then they are parallel to each other. (p. 141)
- Proofs on p. 143, Exercise 3; p. 179, Exercise 37

Theorem 3-10
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other. (p. 141)
- Proofs on p. 141; p. 143, Exercise 12; p. 179, Exercise 38

Theorem 3-11
In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other. (p. 142)
- Proof on p. 143, Exercise 11

(Chapter 3 continued)

Theorem 3-12

Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180. (p. 147)

- Proof on p. 147

Theorem 3-13

Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles. (p. 149)

- Proof on p. 152, Exercise 35

  Corollary

  The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles. (p. 290)

  - Proof on p. 290

Parallel Postulate

Through a point not on a line, there is one and only one line parallel to the given line. (p. 154)

Spherical Geometry Parallel Postulate

Through a point not on a line, there is no line parallel to the given line. (p. 154)

Theorem 3-14

Polygon Angle-Sum Theorem

The sum of the measures of the angles of an n-gon is $(n - 2)180$. (p. 159)

- Proof on p. 163, Exercise 54

Theorem 3-15

Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360. (p. 160)

- Proofs on p. 156 (using a computer) and p. 162, Exercise 46

Slopes of Parallel Lines

If two nonvertical lines are parallel, their slopes are equal. If the slopes of two distinct nonvertical lines are equal, the lines are parallel. Any two vertical lines are parallel. (p. 174)

- Proofs on pp. 387–388, Exercises 42, 43

Slopes of Perpendicular Lines

If two nonvertical lines are perpendicular, the product of their slopes is -1. If the slopes of two lines have a product of -1, the lines are perpendicular. Any horizontal line and vertical line are perpendicular. (p. 175)

- Proofs on p. 346, Exercise 29 and p. 353, Exercise 41

Chapter 4 - Congruent Triangles

base of an isosceles triangle (p. 228)
base angles of an isosceles triangle (p. 228)
congruent polygons (p. 198)
corollary (p. 229)
CPCTC (corresponding parts of congruent triangles are congruent) (p. 221)
hypotenuse (p. 235)
legs of a right triangle (p. 235)
legs of an isosceles triangle (p. 228)
vertex angle of an isosceles triangle (p. 228)

Theorem 4-1

If the two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent. (p. 199)

- Proof on p. 202, Exercise 45

Postulate 4-1

Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent. (p. 205)

Postulate 4-2

Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (p. 206)

Postulate 4-3

Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent. (p. 213)

Theorem 4-2

Angle-Angle-Side (AAS) Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent. (p. 214)

- Proof on p. 214

Theorem 4-3

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (p. 228)

- Proofs on p. 229; p. 231, Exercise 15

  Corollary

  If a triangle is equilateral, then the triangle is equiangular. (p. 230)

  - Proof on p. 231, Exercise 18

Theorem 4-4

Converse of the Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite the angles are congruent. (p. 228)

- Proof on p. 231, Exercise 16

  Corollary

  If a triangle is equiangular, then the triangle is equilateral. (p. 230)

  - Proof on p. 231, Exercise 18

Theorem 4-5

The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base. (p. 228)

- Proof on p. 232, Exercise 29

Theorem 4-6

Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. (p. 235)

- Proof on p. 235

Chapter 5 - Relationships Within Triangles

altitude of a triangle (p. 275)
centroid (p. 274)
circumcenter of a triangle (p. 273)
circumscribed about (p. 273)
concurrent (p. 273)
contrapositive (p. 280)
~~**coordinate proof** (p. 260)~~
distance from a point to a line (p. 266)
equivalent statements (p. 281)
incenter of a triangle (p. 273)
indirect proof (p. 281)
indirect reasoning (p. 281)
inscribed in (p. 273)
inverse (p. 280)
median of a triangle (p. 274)
midsegment (p. 259)
negation (p. 280)
orthocenter of a triangle (p. 275)
point of concurrency (p. 273)

Theorem 5-1

Triangle Midsegment Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side, and is half its length. (p. 260)

- Proof on p. 260

Theorem 5-2

Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. (p. 265)

- Proof on p. 269, Exercise 40

Theorem 5-3

Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. (p. 265)

- Proof on p. 269, Exercise 41

Theorem 5-4

Angle Bisector Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle. (p. 266)

- Proof on p. 269, Exercise 43

Theorem 5-5

Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector. (p. 266)

- Proof on p. 269, Exercise 44

Theorem 5-6

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices. (p. 273)

- Proof on p. 277, Exercise 29

Theorem 5-7

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides. (p. 273)

- Proof on p. 277, Exercise 30

Theorem 5-8

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side. (p. 274)

- Proof on p. 352, Exercise 35

Theorem 5-9

The lines that contain the altitudes of a triangle are concurrent. (p. 275)

- Proof on p. 352, Exercise 36

Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$. (p. 289)

- Proof on p. 289

Theorem 5-10

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side. (p. 290)

- Proof on p. 294, Exercise 33

Theorem 5-11

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle. (p. 291)

- Proof on p. 291

Theorem 5-12

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side. (p. 292)

- Proof on p. 294, Exercise 41

Chapter 6 - Quadrilaterals

base angles of a trapezoid (p. 336)
consecutive angles (p. 312)
isosceles trapezoid (p. 306)
kite (p. 306)
midsegment of a trapezoid (p. 348)
parallelogram (p. 306)
rectangle (p. 306)
rhombus (p. 306)
square (p. 306)
trapezoid (p. 306)

Theorem 6-1
Opposite sides of a parallelogram are congruent. (p. 312)
- Proofs on p. 312; p. 317, Exercise 35

Theorem 6-2
Opposite angles of a parallelogram are congruent. (p. 313)
- Proof on p. 317, Exercise 36

Theorem 6-3
The diagonals of a parallelogram bisect each other. (p. 314)
- Proofs on p. 314; p. 350, Exercise 2

Theorem 6-4
If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. (p. 315)
- Proof on p. 318, Exercise 52

Theorem 6-5
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 321)
- Proof on p. 321

Theorem 6-6
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 321)
- Proof on p. 325, Exercise 12

Theorem 6-7
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (p. 322)
- Proof on p. 322

Theorem 6-8
If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram. (p. 322)
- Proof on p. 325, Exercise 18

Theorem 6-9
Each diagonal of a rhombus bisects two angles of the rhombus. (p. 329)
- Proof on p. 329

Theorem 6-10
The diagonals of a rhombus are perpendicular. (p. 330)
- Proof on p. 333, Exercise 22

Theorem 6-11
The diagonals of a rectangle are congruent. (p. 330)
- Proof on p. 330

Theorem 6-12
If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus. (p. 331)
- Proof on p. 334, Exercise 39

Theorem 6-13
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (p. 331)
- Proof on p. 333, Exercise 23

Theorem 6-14
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (p. 331)
- Proof on p. 334, Exercise 40

Theorem 6-15
The base angles of an isosceles trapezoid are congruent. (p. 336)
- Proof on p. 340, Exercise 38

Theorem 6-16
The diagonals of an isosceles trapezoid are congruent. (p. 337)
- Proofs on p. 337; p. 350, Exercise 3

Theorem 6-17
The diagonals of a kite are perpendicular. (p. 338)
- Proof on p. 338

Theorem 6-18
Trapezoid Midsegment Theorem
(1) The midsegment of a trapezoid is parallel to the bases.
(2) The length of a midsegment of a trapezoid is half the sum of the lengths of the bases. (p. 348)
- Proof on p. 349, Quick Check 1